Selling Capacity Over Time when the Firm is Uncertain of what Customers Know

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Abstract

Perishable capacity is often sold before it is used (e.g., tickets sold weeks before a sporting event) which creates the opportunity to include in the pricing mechanism a recourse strategy, i.e., allowing the firm or buyer to change ownership after an initial transaction. For example, a buyer could be allowed to resell the purchased unit to another buyer (e.g., a ticket exchange), or the firm could offer to refund the buyer if the buyer prefers to relinquish the unit, or the firm could overbook, i.e., sell its capacity twice, possibly denying service to the first buyer (e.g., common practice among airlines). Recourse mechanisms tend to be controversial, both in terms of whether sellers should support them and how they impact buyer welfare. We find that recourse strategies are able to substantially increase the firm's profit and can at the same time increase buyer welfare. Among recourse mechanisms, reselling with sellers posting willingness-to-sell prices is optimal for the seller even though consumers are able to sell for more than they paid. In fact, it is in the interest of the seller to minimize transaction fees to encourage reselling, and the opportunity to resell also benefits consumers. But when conditions for reselling are not ideal, overbooking can be nearly as effective. We conclude that a firm selling capacity in advance should generally adopt some recourse strategy.

1 Introduction

Many firms sell perishable capacity to consumers, capacity that bundles a service with a particular moment in time. Examples include airline flights, hotel rooms, cruise ships, sporting events, music concerts, theatrical events and many others. In these markets consumers learn their preferences over time and often are aware of their preferences well in advance of the time of delivery: e.g., a person may know in January that she has an interest to take a cruise the second week of August. In fact, consumers may value making a transaction commitment well in advance of product delivery: e.g., if a cruise can't be booked for the second week of August, she might prefer in January to make alternative plans for her summer holiday. Hence, there are two key decisions for the firm to make when designing the transaction terms it offers consumers: (a) how does

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the firm price its capacity over time (e.g., is the price for the cruise the same in January as in June) and (b) what options are available to the firm or to consumers if they "change their mind", i.e., what recourse does the firm and/or consumers have if after making the initial commitment (e.g., a cruise ship booking), they want to modify their agreement before the delivery date? Those two decisions are the focus of this paper: we define several options for the firm and determine under which settings the firm should implement them.

Roughly speaking, the firm's price path decision is a choice between one of two strategies. With the "price skimming" strategy the firm implements a decreasing price path over time. The hope is that an early sale is made at a high price, but if that sale does not materialize, the firm lowers the price near the end of the horizon to encourage some transaction. In effect, the firm uses the declining price path to try to "skim off" the consumer with the highest value for the good. The key limitation of this approach is that consumers may be patient - if they are able to wait for the lower price and are confident of availability at the lower price, then a sale is not made at the high price even if a consumer's value exceeds that price (e.g, Coase 1972). The alternative to price skimming is "advance selling", which generally involves an increasing price path and has been shown to be remarkably effective (e.g., Gale & Holmes 1993). In the basic model consumers can purchase from the firm either in the advance period 1 (well before the service delivery) or in the spot period 2 (just before the product is offered). In the advance period consumers have some sense of their preferences, but they are not sure (e.g., in April you might want to book a room with a hotel for an evening in September). They only become sure of their preference during the spot period (e.g., you only know for sure that you want a room at a hotel shortly before the day reserved). Given the uncertainty over their preferences, consumers purchase in advance only if given an inducement to do so. For example, if the firm gives a large enough discount in the advance period, the consumer may take on the risk of buying the product before knowing how much she ultimately likes it. But how does a firm benefit from "selling for less"? In this situation it is indeed possible to "make it up in volume" - the firm can prefer a sure sale for less over an uncertain sale for more.

Although advance selling is promising, it too has limitations. In all the previous analyses of advance selling it is implicitly assumed that the firm knows precisely when consumers switch from being uncertain regarding their preferences (period 1) to when they know their preferences for sure (period 2). It is as if a hotel selling a room for a September evening knows exactly when, say June 1, all consumers learn their value for that room on that fall evening. The problem with this assumption is that it allows the firm to precisely time its price change to the moment when consumers' learn new information regarding their preferences. This is clearly unrealistic and a non-trivial issue for the effectiveness of advance selling - an advance purchase discount does not work well for the firm if consumers happen to know their value for the good when the discount is offered.

The second decision for the firm is to explicitly define what happens if circumstances change (for the consumer or the firm) that may warrant a modification to the initial terms. Rather than leaving these situations to some ad-hoc renegotiation process, the firm and consumers can plan for these contingencies by explicitly including recourse options into the initial agreement, thereby allowing both parties to anticipate correctly the possible outcomes after the initial agreement. We consider the three possible recourse strategies related to who owns the good: reselling, refunds and overbooking.[1] (A fourth, options, is shown to be equivalent to refunds in our model.) With reselling the firm allows the initial buyer to sell the unit to another consumer. With refunds the initial buyer can return the unit to the seller, who then can attempt to sell the unit to another consumer. With overbooking the firm attempts to find a second buyer who is willing to pay more than the initial buyer. If that happens, then the firm takes back the unit from the initial buyer and transfers ownership to the second buyer.

The combination of different price paths and recourse strategies leaves the firm with many possible decisions. It is useful to define a framework for understanding the firm's options. At a high level, the firm's first goal is to generate value in the system. There are two means for generating value: (i) transfer ownership of the unit to some consumer because consumers receive some (though, maybe not much) value from the unit whereas the firm surely receives zero value from owning the unit at the end of the horizon; and (ii) conditional that a transfer is made, transfer the unit to the consumer who values it the most. Given some value is generated, the second goal of the firm is to extract some of that value for itself, i.e., to earn revenue. Unfortunately, there is tension among these goals, i.e., a selling mechanism cannot simultaneously maximize all forms of value generation and value extraction. For example, Myerson (1981) demonstrates that in a one period model the seller's optimal mechanism deliberately does not maximize the probability of a transaction (i.e., limits value creations) so as to increase the fraction of value the firm can extract - the firm is willing to risk not making a transaction because this increases the share of value the firm can earn if there is a transaction. Issues also arise when capacity is sold over time - to increase the probability of a transaction it is tempting to sell to the first willing buyer, but this may reduce the chance the buyer with the highest value actually uses the product.

The two price path strategies, advance selling and price skimming, take different approaches to value creation and value extraction. Advance selling emphasizes the probability of some transaction and the fraction of value extracted at the expense of possibly not allocating the unit to the customer with the highest value. Price skimming emphasizes the probability the consumer with the highest value receives the

[1] While we include all combinations of recourse mechanisms involving a transfer of ownership, there are possibly other recourse mechanisms that do not involve ownership. For example, with a price matching guarantee the firm agrees to change the price paid after some information is learned (Lai *et al.* 2010, Huang *et al.* 2017). In our model there is no need/justification for such price adjustments. Alternatively, there could be a change in the quality of service offered to the customer (Biyalogorsky *et al.* 2005), such as a room or seat upgrade, but we do not include multiple types of products in our model.

good, and accepts the risk that a transaction might not occur.

Recourse strategies are effective because they can both increase the probability a transaction is made and the probability the highest value consumer uses the product. But they do so in different ways. But they do so in different ways. With reselling the initial buyer knows her valuation and therefore sells the unit only if doing so makes her better off. Hence, a second transaction, if it happens, guarantees an increase in generated value. Furthermore, in anticipation of the possible benefits from reselling, reselling increases the likelihood of a transaction for any given price. Refunds make a transaction more likely because it provides the initial buyer with some insurance - if the buyer learns that the product is not very valuable to her, she can return it to the firm for at least a partial refund. Refunds create a floor on the value the buyer can receive. Overbooking is more complex. It may increase the chance of an initial transaction if the firm provides sufficient compensation when service is denied to the initial buyer. It may also increase the chance that the highest value consumer gets the unit because the firm denies service to the initial buyer only if the second buyer is willing to pay a substantial amount. However, because the firm does not know the initial buyer's value, overbooking can make a mistake - the initial buyer might have the highest value and yet be denied service.

To preview our results, we find that advance selling is not always the firm's best choice even when consumers are uncertain of their preferences in the advance period. In particular, whether the advance selling or price skimming is optimal depends on the probability consumers are informed or uninformed of their preferences in advance. If it is likely that consumers know their preferences in advance (e.g., they know in January they want to take a cruise in September), then price skimming is the better strategy. Price skimming is also more desirable as capacity becomes more restrictive (i.e., when there is ample demand relative to capacity) because in those situations there is little risk the capacity goes unused, so it is more important to focus on the proper allocation of the unit (which favors price skimming). No matter whether advance selling or price skimming is optimal, we find that recourse strategies substantially increase the firm's revenue. However, there is considerable variation in the effectiveness of the recourse strategies: reselling is best for the firm (with consumers posting offer-to-sell prices), followed by overbooking and then refunds. In fact, reselling is either optimal or nearly optimal among all possible mechanisms. Finally, even consumers can be better off when recourse mechanisms are implemented because recourse mechanisms tend to increase the amount of value generated (which benefits consumers) rather than enabling the firm to extract a greater share of the generated value (which would harm consumers).

2 Related Literature

There is a large literature focused on selling capacity over time. Our work is distinctive because we integrate into a single model many mechanisms that have previously been treated separately. For example, there are papers on price skimming or advance selling, but not both. There are models on various recourse mechanisms, but none that compare across mechanisms. Thus, they do not present a theory for the reasons why one mechanism is better than another and under what conditions.

A firm that sells multiple non-perishable units to heterogenous consumers who are all aware of their preferences may wish to implement a price skimming strategy - start with a high price to sell to the high value consumers and later lower the price, possibly in stages, to sell to the consumers with lower values. Unfortunately, this decreasing price path is ineffective for the firm if consumers are perfectly forward looking and patient (Coase 1972, Stokey 1981). However, some version of price skimming can be implemented even with forward looking consumers if they are more impatient than the firm (Besanko & Winston 1990) or if early consumers may be rationed due to limited capacity (e.g., Aviv & Pazgal 2008, Liu & van Ryzin 2008, Cachon & Swinney 2009). In our model price skimming is an option for the firm because consumers differ in their delay costs, with some acting myopically due to a high delay cost, and others able to be forward looking due to low (zero) delay cost. Nevertheless, due to restrictive capacity, the forward looking consumer incurs rationing risk. In all of these papers, once a sale is made, it is final - none consider recourse mechanisms.

With non-perishable capacity (e.g., selling durable goods) consumers are able to wait to resolve their preferences before their purchase decision. The luxury of waiting is not always feasible with perishable capacity. This creates the possibility to sell to consumers before there is certainty in their preferences, albeit generally with an initial discount price that is later increased.[2] This strategy, which is usually called advance selling, is not only feasible, it can be highly effective for the firm: Gale & Holmes (1993) show that advance selling allows a monopolist firm to price discriminate between consumers who are relatively indifferent across products (e.g., peak and off-peak flights) and those that have stronger preferences; Dana (1998) shows that advance purchase discounts can arise in a competitive market; DeGraba (1995) demonstrates that a firm can be better off selling a limited amount of capacity in advance to consumers unsure of their preferences; Xie & Shugan (2001) emphasize that advance selling can be effective even with ample capacity; Chu & Zhang (2011) finds that it is always in the firm's interest to sell to consumers with less than perfect preference information; and Cachon & Feldman (2011) show that advance selling via subscriptions can be effective even in services prone to congestion, despite the limited ability of subscriptions to control congestion. (However, Xie & Shugan (2001) and Prasad *et al.* (2011) show that advance selling is not optimal if marginal costs are high

[2] The price path can optimally increase (e.g., Stamatopoulos *et al.* 2018) or decrease (e.g., Golrezaei *et al.* 2017) over time due to time varying (but known) preferences or inventory holding cost management.

and Cachon & Feldman (2017) show that advance selling can harm firms by increasing the competitiveness of the market.) As mentioned earlier, these papers assume that the firm (or firms) know when preferences are revealed to consumers and they do not consider recourse mechanisms.

Among recourse mechanisms, reselling has drawn the most attention. Early work focuses on reselling by individuals who do not value the firm's good, such as ticket scalpers and speculators. These resellers have been generally viewed to be undesirable for the firm. For example, when late arriving consumers have higher valuations than early consumers, a firm might want to sell with an increasing price path. But Courty (2003a) argues that speculators prevent the firm from implementing that strategy because they create competition to sell to the high value consumers. In Courty (2003b) speculators neither help nor harm the firm because they sell at the same price as the firm. More recent work suggests that speculators can benefit a firm that is assumed to have restrictions imposed on its ability to modify its price: in Su (2010) speculators indirectly allow a firm to lower its price when market demand is weak (which isn't what is typically thought of as speculative behavior), and in Cui *et al.* (2014) speculators serve as a low-cost vehicle to transfer units from consumers with low value to consumers with high value. In our model there are no restrictions on what prices can be charged, so speculators play no role (i.e., they are unable to enter and earn a profit). In general, it is not clear why speculators should play a major role in an efficient market. Like the firm, speculators have zero value for the good, and therefore face a disadvantage in the resale market relative to a seller that does value the good (i.e., a consumer). And speculators are likely to have inferior market data relative to the firm for setting appropriate prices. Hence, speculators are more likely to exist in markets with significant trading frictions. The availability of inexpensive information technology has likely reduced these frictions, thereby enabling efficient consumer-to-consumer reselling exchanges (e.g., StubHub).

As in our model in which consumers arrive sequentially, Yang *et al.* (2017) consider reselling positions in a queue. However, consumers in their model do not learn information over time regarding their valuation and they do not consider dynamic pricing. Nevertheless, in their setting they demonstrate that social welfare and firm profits can increase substantially by allowing consumers to resell.

Some work considers refunds and options. Xie & Gerstner (2007) and Gallego & Sahin (2010) study a monopolist selling to consumers over two periods. In the first period consumers are uncertain of their preferences but their preferences are revealed to them in the second period. With a refund a consumer pays the full price in period 1 but can receive a partial refund in period 2 if the consumer's value is low. Equivalently, this can be implemented using options - the consumer pays a non-refundable fee in period 1 for the option to purchase, and an exercise fee in period 2 if the consumer decides to purchase. Both papers show that refunds/options can increase the seller's revenue but neither considers alternative recourse mechanisms. Guo (2009) extends Xie & Gerstner (2007) to a competitive setting and demonstrates that refunds may no longer be offered, thereby suggesting that competition is a reason for the limited use of refunds in practice. We offer an alternative explanation for the narrow application - refunds are the least effective of the recourse mechanisms for the firm.

Overbooking is the practice of selling beyond capacity: e.g., selling more tickets than seats on a flight, or more reservations for a hotel than rooms, or scheduling more appointments in a day than a doctor could actually deliver. Most research on overbooking focuses on how much to sell beyond capacity, which is not the focus of this paper and some concentrate on the pricing decision (e.g., Weatherford & Bodily 1992, Biyalogorsky *et al.* 1999, Karaesmen & Van Ryzin 2004, Gallego *et al.* 2008). Consumer or social welfare are generally not considered, nor is overbooking compared to other recourse mechanisms.

3 Model Description

We study a model in which a firm sells perishable capacity to consumers, such as admission to some event, a ticket on some form of transportation, or a room at a type of lodging. Demand is uncertain and capacity is potentially restrictive (i.e., demand may exceed supply). The capacity is used at a particular point in time, and consumers can anticipate ahead of that time their need for the capacity. However, they learn over time the strength of their preference for the capacity, what we refer to as the consumer's value. For example, some people may precisely know their value for a concert well in advance of the event, whereas other only know they will have some value and the specific value is learned closer to the time of the event. Because consumers can anticipate their value for the capacity, the firm can sell this capacity over time, e.g., well in advance or closer to "on the spot" (i.e., just before when the capacity is used). As a result, many selling mechanisms are feasible. The remainder of this section details the specifics of the model.

A single firm sells one unit of capacity over a two period horizon, the advance period 1 and the spot period 2. The firm dynamically posts a take-it-or-leave-it price – an advance price in period 1, p_1, and a spot price in period 2, p_2. The unit is used (if purchased by some consumer) at the end of period 2. The firm incurs zero marginal cost to deliver the unit. If the unit is not purchased by a consumer over the two periods, then the capacity is wasted, i.e., the firm receives no value for unsold capacity.

The market consists of two rational buyers, buyer A and buyer B. Their values, $V_{i\in\{A,B\}}$, for the unit are independent and uniformly distributed on the interval $[0, 1]$. Let $v_{i\in\{A,B\}}$ be the realization of their value. Buyer A arrives to the market in the advance period either "informed" or "uninformed". An informed buyer A knows v_A upon arrival in period 1 whereas an uninformed buyer A does not yet know v_A. For example, say the unit is a seat at a sporting event. Buyer A may be certain of the value of that seat in advance (i.e., informed), or merely knows that attending the event may be desirable (uninformed) but is not surely desirable due to outcomes of other uncertain events. While buyer A knows whether he/she is informed or not, the firm only knows with probability β that buyer A is informed. Buyer B arrives to the market at the start of the spot period 2. All buyers present in period 2 are aware of their value for the unit because it is close enough to the time when the capacity is used. (See Papanastasiou & Savva (2017) and Feldman *et al.* (2019) for models in which consumer learning is endogenously determined by the firm's actions rather than, as in our model, an exogenous process.) Given that consumer values are identically distributed, differences in pricing across time cannot be attributed to changes in value (changes in value over time play a critical role in Su (2007)).

The parameter β is an important feature of this model. In many markets some consumers may know enough about the product to assess their value early on, while other consumers may consider purchasing early without having this precise knowledge. More importantly, while individual consumers know whether they are ignorant or not, the firm generally is not able to distinguish between consumers, and therefore cannot tell whether it is offering the product to informed or uninformed consumers. The literature has not considered this plausible situation. In the literature, consumers in the advance period are either informed ($\beta = 1$ is assumed) or uninformed ($\beta = 0$ is assumed) and this is known by all. As we later demonstrate, the firm's inability to know precisely what consumers know strongly influences which selling mechanism should be used and the revenues earned.

The firm offers to sell the unit of capacity to buyer A in period 1 for the price p_1. No matter whether the buyer is informed or uniformed, buyer A can purchase the unit in period 1. However, if the informed buyer A does not purchase in period 1, then the buyer exits the market. In effect, the informed buyer A's value for the unit deteriorates over time if the buyer does not have assurance of access to the unit. For example, an informed buyer A may value attending a basketball game to celebrate with his daughter her birthday, but only if it is possible to make plans to attend. If the buyer is unable to acquire the seat in advance, then the buyer prefers to make alternative plans (to celebrate her birthday). (Courty (2003a) uses a similar preference.) In contrast, the uninformed Buyer A remains unsure of v_A in the advance period, but knows that v_A is observed at the start of period 2 and that it is indeed the value for using the capacity at the end of period 2. For example, the buyer may want to celebrate his daughter's birthday at a basketball game, is unsure (in advance) if she will be able to attend, but knows that this uncertainty is resolved later (in the spot period).

If both buyers are present in period 2 and the firm is still offering the unit for sale, then buyer B is given the opportunity to purchase the item before buyer A. Consequently, an uninformed buyer A is first given an opportunity to purchase the item in period 1 (before v_A is known) and then has a second opportunity in period 2 (after v_A is observed) but only after buyer B is given an opportunity to purchase in period 2.

Hence, delay is costly - if buyer A chooses to not purchase in period 1, then the buyer risks not being able to purchase the unit in period 2 even if the buyer discovers a high valuation.

The parameters and sequence of events are common knowledge to the buyers and the firm. All agents are risk-neutral, utility maximizers and correctly anticipate future actions. The firm's objective is to design the terms of trade to maximize expected revenue (which is equivalent to expected profit given the zero marginal cost for delivering capacity).

The interesting strategic interaction in this model occurs between the firm and buyer A. (With only buyer B in the market the optimal selling mechanism is a straightforward posted price because buyer B knows v_B and there is only a single selling period.) While there are many selling mechanisms, the ones considered in this paper can be characterized by the (a) recourse strategies allowed and (b) purchase timing.

A recourse strategy specifies what can be done after an initial transaction agreement. Four recourse strategies are considered in this paper: none, reselling, refunds/options and overbooking. With the "no recourse" strategy there is no recourse, i.e., once a buyer purchases the unit, they own the unit, which means only they can use it. Reselling allows a buyer to resell the unit to another buyer. Refunds/options allows a buyer to return the unit to the seller at a pre-specified price. The buyer may want to do this upon learning of a low valuation for the unit. If this is done early enough, the seller can then try to sell to another buyer. Finally, overbooking allows the firm to sell its one unit of capacity twice. To be specific, after selling the unit to one buyer, overbooking allows the firm to try to sell it to another buyer, and if that second buyer takes the unit, the firm buys back the unit from the original buyer and denies service.

Each of the four recourse strategies can come in two forms. With an "advance selling" strategy the firm induces the uniformed buyer A to purchase in period 1. Given that buyer A is uninformed in period 1, this approach often (but not always) involves an increasing price path over time - buyer A needs a discount to buy in advance before v_A is known. Alternatively, the firm could choose a mechanism in which the uniformed buyer A purchases only in the spot period. This generally leads to a decreasing price path over time, so we refer to this as a "price skimming" strategy - the firm uses its prices to try to "skim off" the consumer with the highest value.

With each mechanism we evaluate the firm's optimal contract offer, the firm's optimal revenue, and the expected total surplus (i.e., social welfare), which is the expected value generated by the capacity. Surplus depends on two factors: (i) the probability the unit is transferred to a buyer (no value is generated unless a transfer is made) and (ii) the probability it is consumed by the buyer with the highest value (more value is generated if the consumer with the higher value gets the unit). Prices affect surplus only indirectly by changing these probabilities.

We consider many extensions to this base model, which can be found in the appendix. Specifically, we study the following variations: (i) Optimal auction in the second period: in the main model we focus on seller posted second period prices even though an optimal auction (second price auction with a reserve price) is the optimal single period mechanism (Myerson 1981), because of the challenge in implementing an auction. Nevertheless, we analyze an equivalent model in which the firm runs an optimal auction in Appendix B and refer to these results when appropriate. (ii) Informed buyer A is patient: in the main model, we assumed that an informed buyer A that does not buy in period 1, leaves the market. It is possible to consider an alternative model in which the informed buyer A is patient and considers purchasing in period 2. That is, say δ is the discount factor. The base model assumes that an informed buyer has $\delta = 1$ and an uninformed buyer has $\delta = 0$. The alternative model assumes that both buyers have $\delta = 1$. The firm weakly prefers buyer A to be patient, because having a potentially bigger market size in period 2 is better than a smaller one. Appendix C.1 provides the details of this alternative model. (iii) Random priority rule: the main model assumes that if the firm has a unit in period 2, buyer B gets priority over buyer A. Any other allocation rule leads to qualitatively similar results. The only requirement is for the rule to be common knowledge. Appendix C.2 analyzes a model with a random allocation rule.

4 Resale and The Optimal Mechanism

Traditionally having a bad reputation, online marketplace companies such as Stubhub (owned by eBay), Ticketmaster (owned by Live Nation), RazorGator and many others have made the exchange of tickets through reselling safer and more efficient. The result has been a rapidly growing market which is expected to increase in value to about $15 billion by 2020 (Technavio 2015).

In the resale recourse mechanism the firm allows a buyer to resell a purchased unit to another consumer. To be specific, if buyer A purchases the unit in the advance period then buyer A may attempt to resell it to buyer B in the spot period, period 2, via a take-it-or-leave-it price offer, p_r. No matter whether A was informed or uninformed when the unit was purchased in period 1, because A is posting the resale price in period 2, A knows the value v_A for the unit before deciding on the resale price, p_r. For simplicity, there are no transaction costs.

The option to resell influences the buyer's expected utility. Let $\mathcal{R}_A(p_r)$ be buyer A's utility conditional on owning the unit as a function of the chosen resale price:

$$\mathcal{R}_A(p_r) = p_r v_A + (1 - p_r) p_r.$$

Buyer A keeps the unit and earns v_A if $v_B < p_r$ and resales to buyer B at p_r, otherwise. To maximize expected utility, buyer A chooses the resale price $p_r^* = (1+v_A)/2$ and earns expected utility $\mathcal{R}_A^*(v_A) = (1+v_A)^2/4$.

In period 1 the informed buyer A earns utility $\mathcal{I}_1 = \mathcal{R}_A^* - p_1$ from purchasing the unit and the buyer purchases the unit if $\mathcal{I}_1 \geq 0$ (because the alternative is to not purchase at all). Thus, the informed buyer A purchases in period 1 if $v_A \geq \tilde{v}_A$, where $\tilde{v}_A = \max\left\{0, 2\sqrt{p_1} - 1\right\}$. Due to the option to resell, an informed buyer A is willing to purchase in the advance period even if the buyer incurs an initial loss, i.e., $\tilde{v}_A < p_1$. The option to resell also influences the uniformed buyer's utility. This buyer's expected utility from purchasing the unit in period 1 is

$$\mathcal{U}_1 = \int_0^1 \mathcal{R}_A^*(x)\,dx - p_1 = \frac{7}{12} - p_1.$$

The firm can adopt one of two pricing strategies–an advance selling strategy or a price skimming strategy. With the advance selling strategy the firm sets prices such that the uniformed buyer purchases in period 1. If the firm fails to sell the unit in period 1, then only buyer B remains as a possible customer in period 2. Thus, the firm's optimal spot period price is $p_2^* = 1/2$ and the optimal spot period revenue is $\Pi_2^* = 1/4$.

While with advance selling the uninformed buyer A is expected to purchase in period 1, the buyer does have the option to wait to try to purchase in period 2 (off equilibrium). Doing so yields expected utility $\mathcal{U}_2$, where

$$\mathcal{U}_2 = p_2\left(\int_{p_2}^1 (x - p_2)\,dx\right) = \frac{1}{16}.$$

The uninformed buyer A purchases in the advance period if $\mathcal{U}_1 \geq \mathcal{U}_2$, which requires $p_1 \leq 7/12 - 1/16 = 25/48$.

The firm's period 1 revenue function with advance selling (assuming the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint is satisfied) is

$$\Pi_1 = (1 - \beta\tilde{v}_A)\,p_1 + \beta\tilde{v}_A\Pi_2^*.$$

The first term is revenue from selling in period 1 to an uninformed buyer A or an informed buyer A with sufficiently high v_A. The second term is the revenue earned conditional that the buyer is informed with low value.

With the price-skimming strategy only the informed buyer A considers a purchase in period 1, and does so if the buyer's value exceeds the threshold $\tilde{v}_A = \max\left\{0, 2\sqrt{p_1} - 1\right\}$. This threshold influences $\omega(\tilde{v}_A)$, the probability buyer A is in the market in period 2 conditional that the firm did not make a sale in period 1:

$$\omega(\tilde{v}_A) = \frac{1-\beta}{1-\beta+\beta\tilde{v}_A} \tag{1}$$

The firm's revenue function in period 2 (conditional on still owning the unit) is

$$\begin{aligned}\Pi_2 &= \omega\left(\tilde{v}_A\right)\left(1-p_2^2\right)p_2+\left(1-\omega\left(\tilde{v}_A\right)\right)\left(1-p_2\right)p_2 \\ &= p_2\left(1-p_2\right)\left(1+\omega\left(\tilde{v}_A\right)p_2\right).\end{aligned} \tag{2}$$

With probability $\omega\left(\tilde{v}_A\right)$ the firm is posting a price, p_2, to sell to two consumers (B and A), and with probability $1-\omega\left(\tilde{v}_A\right)$ the firm is posting a price to sell only to one buyer (B). The period 1 revenue function with price skimming is

$$\Pi_1 \quad = \quad \beta(1-\tilde{v}_A)p_1+\left(1-\beta+\beta\tilde{v}_A\right)\Pi_2.$$

The first term is revenue from the informed buyer A in period 1 and the second term is revenue from period 2 given no sale in period 1.

Theorem 1. *With the reselling mechanism there exists a unique β^r, such that*

1. *If $\beta \leq \beta^r$, the firm follows an advance selling strategy with prices $p_1^* = 25/48$ and $p_2^* = 1/2$. The firm's optimal revenue, $\Pi_1^* = \left(150-\left(65\sqrt{3}-78\right)\beta\right)/288$, is decreasing in β. Surplus is $S^* = (5/576)\left(72-\left(37\sqrt{3}-60\right)\beta\right)$*

2. *Otherwise, the firm follows a price skimming strategy with $p_1^* = \left(1+\tilde{v}_A^*\right)^2/4$,*

$$p_2^* = \frac{-1+\omega+\sqrt{1+\omega+\omega^2}}{3\omega}$$

$$\tilde{v}_A^* = \frac{1}{3}\left(-1+2\sqrt{1+3p_2^*\left(1-p_2^*\right)}\right)$$

and ω is given by (1). The firm's revenue, Π_1, is increasing in β. Surplus is

$$S^* = \frac{1}{2}\left(\left(1-p_2^*\right)^2+\beta\left(2p_1^*\left(1-\sqrt{p_1^*}\right)-2\sqrt{p_1^*}p_2^{*2}+p_2^*\left(-1+2p_2^*+p_2^{*2}\right)\right)\right)$$

An advance selling strategy usually involves an increasing price path over time, because the firm needs to provide uninformed buyers an incentive to take the risk and purchase in advance. Theorem 1 highlights that with reselling firm's prices actually decline somewhat even with advance selling: $1/2 = p_2 < p_1 = 25/48$. The uninformed buyer is willing to pay the higher period 1 price because the buyer is aware of the opportunity to potentially resell the item to buyer B. This provides buyer A with some insurance in case of a poor realization for V_A : if the buyer learns that the unit is not valuable, the buyer posts a "low" period 2 price to increase the odds of salvaging something from the transaction. And if the uninformed buyer A learns of a high value for v_A then the buyer can post an appropriately high price on the resale market. Although the resale price is usually higher than the period 1 price, it might actually be lower if buyer A learns of a very low value, i.e., when $v_A < 1/24$.

With advance selling, both the firm's revenue and surplus are decreasing as buyer A is more likely to be informed (as β increases). Advance selling with reselling is relatively sensitive to the probability that buyer A is informed, β: if the firm expects $\beta = 0$, but in fact $\beta = 1$, then the firm's revenue is 23% lower than it expects.

The price skimming strategy is more profitable as the likelihood that buyer A is informed increases, i.e., as β increases. However, this strategy is relatively insensitive to β : with $\beta = 0$, then $\Pi_1 = \Pi_2 = 2/3^{3/2} \approx 0.3849$, whereas with $\beta = 1$ (the other extreme) the firm's revenue is only 6.0% higher ($\Pi_1 \approx 0.4079$). Note that when buyer A is surely uninformed, i.e. $\beta = 0$, the price skimming strategy is equivalent to a single posted price, because the uninformed waits in a price skimming equilibrium and resale cannot occur.

Taken the two together, we find that in equilibrium the firm adopts an advance selling strategy if it's likely that buyer A is uninformed (β is low) to maximize the chance that a transaction is made. If buyer A is likely informed, however, then the firm chooses to try to "skim" the buyer and prices so that an uninformed buyer waits.

In the considered mechanism buyer A posts a willingness-to-sell price in the resale market. Alternatively, buyer B could post a willingness-to-buy price in the resale market. While the total gains from trade are the same regardless of who posts the price, buyer A earns more surplus from posting a willingness-to-sell price. Consequently, the firm can extract more surplus and strictly prefers reselling with prices posted to sell rather than posted to buy. In theory, with price skimming the firm is indifferent between which buyer posts prices in the reselling market. Both buyers know that in the resale market buyer A's valuation is in the interval $[\tilde{v}_A, 1]$. So buyer B can participate in the resale market only if her value is also in the interval $[\tilde{v}_A, 1]$. Hence, each buyer has the same information about the other buyer, which implies that both buyer A's willingness-to-sell price and buyer B's willingness-to-buy price maximize the gain from trade. Either way, the firm is able to fully extract this gain from the resale market: the indifferent consumer's gain from trade is the same $(1 + \tilde{v}_A)^2/4$ no matter which buyer posts the price and the firm is able to fully extract this gain.

Notwithstanding the previous result, most reselling markets operate in which the customer in possession of the unit posts a willingness-to-sell price rather than the other way around. The firm strictly prefers this with advance selling. However, even if the firm uses a price skimming strategy, sellers posting rather than buyers posting could be preferred if the buyers are not fully rational. When buyer A posts a willingness-to-sell price, it is clear that the posted price should be higher than the buyer's own value, v_A. (In fact, the resale price in this equilibrium is always greater than the firm's period 1 price, $p_1^* < p_r^*$.) Similarly, a fully rational buyer B posts a willingness-to-buy price of $(v_B + \tilde{v}_A)/2$ because the buyer should know that A's value must be at least $\tilde{v}_A$. In actual markets, buyers may not be aware of the prices that previous customers paid or fail to condition on those prices. Thus, they might offer willingness-to-buy prices that are too low. This would reduce surplus and would consequently reduce the firm's ability to earn the gains from the reselling market, leading to a preference for sellers posting prices.

We have assumed that buyer A earns the entire profit from any reselling of the unit. It might be tempting for the firm to try to earn some portion of reselling revenue, but any transaction costs in the reselling market would lower buyer A's earnings and in turn, lower the firm's revenue as well. In fact, the next central result shows that a resale mechanism with no transaction costs is optimal - the firm cannot earn higher revenue from implementing any other mechanism.

Theorem 2. *Resale with zero transaction fees is the optimal mechanism. There exists a unique β^*, so that if $\beta \le \beta^*$, it is optimal for the firm to implement resale with advance selling and a posted price in period 2. Otherwise, it is optimal to implement resale with price skimming and an optimal auction in period 2. The firm's revenue is decreasing in β.*

Theorem 2 indicates that the selling mechanism in period 2 is relevant. In this paper we focus on posted price mechanisms. Posted prices are a natural and commonly observed mechanism because auctions can be costly - they impose delay/uncertainty costs on customers and are non-trivial to administer. However, a second price auction with a reserve maximizes the firm's revenue in period 2. A key difference between posted prices and an auction is that there is never an allocation error with an auction. With posted prices it is possible that buyer B is awarded the unit even though buyer B values it less than buyer A. This would seem to favor the auction, and is partially why an auction maximizes revenue in period 2. And in fact, because price skimming focuses on customers that are not concerned with an allocation error (buyer A in the price skimming equilibrium either purchases in period 1 or finds an alternative), price skimming indeed prefers the revenue maximization feature of the auction. That said, with price skimming the improvement an auction provides over posted prices is relatively small - in the worst case price skimming with reselling and a second period posted price achieves 99.67% of the optimal revenue. Although posted prices somewhat limit the effectiveness of price skimming, with advance selling the allocation risk imposed by posted prices is actually beneficial - buyer A is willing to pay more in period 1 to avoid an allocation error, which works to the firm's advantage. Figure 1 compares the revenues of advance selling and price skimming with resale assuming an optimal auction and a posted price in period 2. (See Appendix B for derivations of the equilibrium results with a second period auction.)

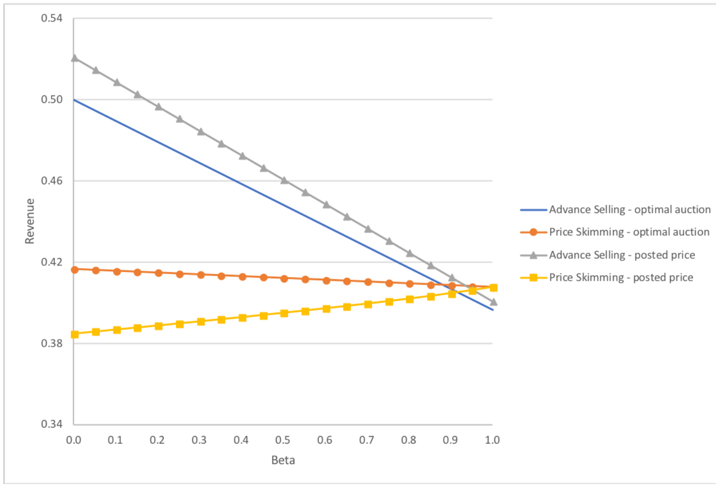


Figure 1. Revenue comparison of resale mechanisms with optimal auction and posted price in period 2.

In contrast to the firm's revenue with a posted price, with a second period auction, the price skimming strategy is less profitable as β increases. The likelihood that buyer A is informed affects the revenue with price skimming in two ways. First, as β increases, the firm can benefit from setting a high price in period 1 to try to "skim" the informed buyer, which increases revenue. Second, an increase in β decreases expected demand in period 2, because an informed buyer that does not purchase leaves the market. The demand reduction negatively affects revenue regardless of the pricing strategy in the second period, but the effect is larger with an auction contributing to an overall decrease in revenue as a function of β.

Not only is reselling optimal, it is optimal to allow customers to transfer units without charging transaction fees. Despite losing the ability to collect fees from transfer, to collect such fees the firm needs to reduce the first period price which harms its overall revenue. In essence, it is better for the firm to earn a higher price in period 1 "for sure", then to earn a transfer fee "with probability" (only if a transfer is indeed made). Therefore, transfer fees exist in practice in markets with inefficiencies (e.g., to cover operating costs) or to sustain a third party that may provide additional benefits that we do not model (e.g., matching capabilities).

While reselling is the optimal mechanism, it is not the only one observed in practice. Many firms do not implement a recourse strategy. Others that do, may choose to allow returns for a partial refund or practice overbooking. We analyze these alternative mechanisms in Sections 5-7 and compare their efficiency to the optimal mechanism. In Section 9, we discuss why despite its optimality, firms do not practice reselling universally.

5 No recourse

A basic mechanism does not allow for recourse - no further transactions or transfers of ownership can occur after a buyer commits to a purchase. In this simple mechanism the firm chooses two prices, p_1 and p_2, one for each period. The firm selects p_2 only after observing the outcome from period 1 (i.e., the firm prices dynamically and does not commit to p_2 in advance).

The informed buyer A only considers purchasing in period 1 and does so if $v_A \geq p_1$. An uninformed buyer A's expected utility of buying in period 1 is

$$\mathcal{U}_1 = \mathbb{E}\left[V_A\right] - p_1 = \frac{1}{2} - p_1$$

and the expected utility of not buying in period 1 is

$$\mathcal{U}_2 = p_2 \int_{p_2}^{1} \left(v - p_2\right) dv = \frac{1}{2} p_2 \left(1 - p_2\right)^2 .$$

where the first term, p_2, is the probability that buyer B does not purchase (giving buyer A an opportunity to purchase in period 2) and the second term is buyer A's expected utility conditional on buyer A's ability to purchase.

With advance selling, the firm prices so that the uninformed buyer A purchases in period 1 and hence $\mathcal{U}_1 \geq \mathcal{U}_2$ must hold. If there is no sale in period 1, it must be that buyer A was informed and has a low value, $v_A < p_1$. In that case, the firm knows that only buyer B remains in period 2. Period 2 revenue is $\Pi_2\left(p_2\right) = p_2\left(1 - p_2\right)$ and maximum revenue $\Pi_2^* = 1/4$ is obtained with $p_2^* = 1/2$. Period 1 revenue (assuming the constraint $\mathcal{U}_1 \geq \mathcal{U}_2$ is satisfied) is

$$\Pi_1\left(p_1\right) = \left(1 - \beta p_1\right) p_1 + \beta p_1 \Pi_2^*.$$

With price-skimming, the firm sets prices so that the uninformed buyer A does not purchase in period 1, i.e., $\mathcal{U}_1 < \mathcal{U}_2$. Hence, a sale occurs in period 1 only if buyer A is informed and has value $v_A \geq p_1$. The absence of a sale in period 1 can occur either because buyer A is uninformed, or because buyer A is informed but with too low a value to purchase, $v_A \leq p_1$. In the former case buyer A remains in the market in period 2, whereas in the latter case the buyer exits the market. Let $\omega\left(p_1\right)$ be the probability buyer A is in the market in period 2 conditional on there not being a sale in period 1:

$$\omega\left(p_1\right) = \frac{1 - \beta}{1 - \beta + \beta p_1} \tag{3}$$

This conditional probability depends on p_1 because the first price influences the informed buyer's probability of purchase. For example, $\omega(0) = 1$ because then the informed buyer A surely purchases (i.e., the failure to sell in period 1 can only mean that the buyer was uniformed), whereas $\omega(1) = 1 - \beta$ because the informed buyer A never purchases.

Period 2 revenue is

$$\Pi_2 = \omega(p_1)\left(1 - p_2^2\right)p_2 + (1 - \omega(p_1))(1 - p_2)p_2.$$

The first term is revenue when both buyers remain in the market and the second term is revenue when only buyer B is present. Period 1 revenue is

$$\Pi_1(p_1) = \beta(1 - p_1)p_1 + (1 - \beta + \beta p_1)\Pi_2$$

The first term is revenue when the informed buyer A purchases in period 1 and the second term is the expected revenue from period 2 when there is no sale in period 1.

Theorem 3. *In the no-recourse mechanism there exists a threshold β^0, such that*

1. *If $\beta \leq \beta^0$, the firm implements an advance selling equilibrium in which the uninformed buyer purchases in period 1. The firm sets prices $p_1^* = 7/16,\ p_2^* = 1/2$. The firm's revenue is $\Pi_1^* = (7/256)(16 - 3\beta)$ and total surplus is $S^* = 1/2 + (35/512)\beta$.*

2. *Otherwise, the firm implements a price-skimming equilibrium. Prices satisfy the following system of equations:*

$$p_2^* = \frac{-1 + \omega + \sqrt{1 + \omega + \omega^2}}{3\omega}$$

$$p_1^* = \frac{1}{2}\left(1 + (1 - p_2^*)\,p_2^*\right)$$

where ω is given by (3). Equilibrium prices are bounded such that $p_2^* \in [1/2, 1/\sqrt{3}]$, and $p_1^* \in \left[\left(2 + \sqrt{3}\right)/6, 5/8\right]$. Optimal revenue Π_1^* is increasing in β and bounded such that $\Pi_1^* \in \left[2/\left(3\sqrt{3}\right), 25/64\right]$. Surplus is $S^* = (1/2)\left(\beta\left(1 - p_1^{*2}\right)(1 + p_1^*) + (1 - \beta)\left(1 - p_2^{*2}\right)(1 + p_2^*)\right)$.

Theorem 3 mimics the structure of Theorem 1 in that the firm implements an advance selling strategy when buyer A is likely uninformed (β is low) and a price skimming strategy when it is likely the buyer A is informed (β is high). The theorem illustrates that as opposed to the reselling mechanism, with advance selling prices are increasing over time, $p_1^* < p_2^*$. Indeed, this is generally the case because the uniformed buyer A requires a discount to compensate for purchasing before v_A is learned (i.e., the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint binds). However, if buyer A is indeed informed of v_A, then the buyer benefits from the lower advance price. Thus, advance selling with no-recourse is most effective for the firm when the buyer is likely to be uninformed of v_A, i.e., Π_1 is decreasing in β. Furthermore, as with the reselling mechanism, advance selling with no-recourse is relatively sensitive to the probability that buyer A is informed, β: implementing this strategy assuming buyer A is uninformed ($\beta = 0$) when buyer A is indeed informed ($\beta = 1$) yields 18.75% lower revenue than expected ($1 - \Pi_1^*(1)/\Pi_1^*(0) = 0.1875$).

Advance selling can also be a risky strategy because it assumes the firm is able to choose prices such that the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint binds. That requires precise measurement of the buyer's value. Suppose the firm were to assume the buyer's value were drawn from a [0,1] uniform distribution but the buyer knows that it is actually drawn from a [0,1-ϵ] uniform distribution, with a positive and small ϵ. Due to the firm's error, even though it can be very small, the uniformed buyer chooses not to purchase in period 1 at price $p_1^* = 7/16$ (because the buyer's expected value is slightly lower than what the firm believes it to be). Without a sale in period 1, the firm then offers $p_2 = 1/2$ in period 2 and earns between 14.3% ($\beta = 0$) and 30.8% ($\beta = 1$) less than what it would have earned in the equilibrium had it not made the error. The firm can hedge against this error by reducing its period 1 price, but doing so clearly makes advance selling less advantageous.

Although an informed buyer is not desirable to the firm with advance selling, it is good for total surplus (i.e., surplus is increasing with β). Total surplus is determined by the value of the party that owns the unit at the end of the horizon (buyers or the firm), i.e., it is concerned with the probability of a transfer and with allocating the unit to the consumer that values it most. The advance selling strategy guarantees a transfer to an uninformed A, but it does nothing to ensure that the unit is allocated to the customer with the highest value. To screen for the buyer with the higher value, it is best if buyer A is more likely to be informed.

With the price skimming strategy prices decline over time because the firm uses that price path to screen the consumers by their values - start high in the hope that buyer A might have a high value, and if not, then lower the price to ensure that some transaction occurs. The limitation of this strategy is that the uniformed buyer A surely is not willing to pay the high initial price, implying that focusing on period 1 revenue, the firm from a high β. However, if the firm does not sell in period 1 it faces decreased demand in period 2 if buyer A is informed (because an informed buyer that does not purchase in period 1, leaves the market). Therefore, whether revenue increase or decrease with β with price skimming depends on the relative magnitude of these effects. Overall, based on Theorem 3, in this case, the firm using a price skimming strategy prefers buyer A to be informed (Π_1 is increasing in β). The two opposing effects imply that the price skimming strategy is a relatively robust strategy in the sense that the optimal period 1 price is insensitive to $\beta : p_1 \in [0.6220, 0.6250]$. Consequently, the firm's optimal revenue is also relatively insensitive to β: $\Pi_1^* \in [0.3849, 0.3906]$.

6 Refunds and Options

With the refund recourse mechanism the firm chooses in period 1 a price, p_1, and a refund amount, $f \leq p_1$. If buyer A purchases the unit in period 1 for p_1 then the buyer has the right to return the unit to the firm and receive a payment of f. The buyer must return the unit at the start of period 2, just after learning the value v_A. If buyer A returns the unit, then the firm can try to sell the unit to buyer B in period 2.

This mechanism is equivalent to an option: buyer A pays in period 1 a non-refundable amount $p_1 - f$ for the option to purchase the unit in period 2, after observing v_A, for the exercise fee f. These outcomes are equivalent to the refund mechanism - if the option is exercised, then the buyer's total cost is p_1, but if not exercised, then the buyer incurs a net loss of only $p_1 - f$. For simplicity, we assume that this mechanism is presented in the form of a refund rather than an option.

The refund mechanism provides no value to the informed buyer A because the buyer only makes the initial purchase when $v_A \geq p_1$. However, the refund mechanism is valuable to the uninformed buyer A because it allows the buyer to receive $\max\{f, v_A\}$ rather than just v_A. While a returned unit is costly to the firm, it at least provides a signal that buyer A is not the buyer with the highest value for the item - a return creates an opportunity to sell to buyer B.

The uninformed buyer A anticipates the value of the refund and expects to earn the following utility from a period 1 purchase:

$$\mathcal{U}_1 = f^2 + (1 - f)\left(\frac{1+f}{2}\right) - p_1 = \frac{1}{2}\left(1 + f^2\right) - p_1 :$$

If his realized value is lower than the refund f, buyer A returns the unit and receives the refund and otherwise, he keeps the unit. Alternatively, the uniformed buyer A can choose not to purchase in period 1 and earn utility $\mathcal{U}_2 = p_2 (1 - p_2)^2 / 2$.

With advance selling the firm starts period 2 with a unit if either an uninformed buyer A purchased a unit in period 1 and returned it or if an informed buyer A did not purchase in period 1. Regardless, the only possible customer in period 2 is buyer B. Hence, the optimal period 2 price is $p_2^* = 1/2$.

The firm's period 1 revenue is

$$\Pi_1 = \beta\left((1 - p_1) p_1 + p_1 (1 - p_2^*) p_2^*\right) + (1 - \beta)\left(p_1 - f^2 + f p_2^* (1 - p_2^*)\right).$$

Refunds do not play a role if Buyer A is informed. If Buyer A is uninformed, in an advance selling equilibrium, Buyer A purchases the unit in period 1 for p_1, but returns the unit and receives a refund f if $v_A < f$, in which case the firm tries to sell the unit to Buyer B at $p_2^* = 1/2$.

Given that the refund provides no value to the informed buyer and this buyer is the only one that consider purchasing in period 1 with price skimming, it follows that this price skimming equilibrium is equivalent to the price skimming equilibrium with the no-recourse mechanism. In fact, the next theorem establishes that the firm always chooses an advance selling strategy when implementing a refund policy.

Theorem 4. *With the refund mechanism the firm implements an advance selling equilibrium for all β. The equilibrium prices are $p_2^* = 1/2$ and $p_1^* = \frac{1}{2}\left(1 + f^2\right) - 1/16$ where f is the unique solution to*

$$2\left(1-\beta\right) - 8\beta f^3 - \left(8 - 11\beta\right) f = 0$$

Surplus is

$$S^* = \frac{1}{512}\left(64\left(4 + 3f^* - 4f^{*2}\right) + \beta\left(35 - 192f^* + 240f^{*2} - 64f^{*4}\right)\right).$$

7 Overbooking

Overbooking is associated with the practice of selling more capacity than is actually available. One reason to do this is because some customers do not use the capacity despite their reservation. In our model this would occur if buyer A does not want to use the unit because she learns her value for the unit is 0. In that case there is no loss of surplus from denying service, so the interesting question is merely by how much to overbook. However, because there is no probability mass on zero in our model, this reason for overbooking does not pertain in our context. Instead, in our model the firm uses overbooking as a method to find the consumer with the highest valuation, albeit at the risk that it may be forced to deny service to a customer who previously purchased a unit. To be specific, when the firm overbooks it begins with an offer of price p_1 to buyer A in period 1. If buyer A does not purchase in period 1, then the firm makes an offer of price p_2 in period 2 to buyer B and then, if still in the market, buyer A. The interesting feature of overbooking occurs if buyer A does purchase in period 1. Despite having sold its capacity to buyer A, the firm nevertheless offers buyer B the unit in period 2 at price p_o. If buyer B does not take the offer, then buyer A consumes the unit. But if buyer B does take the unit, then the firm buys back the unit from buyer A at the price b. The firm announces the buy back price b in period 1 along with the period 1 price p_1. Hence, if buyer A purchases in period 1 then the buyer is aware that service might be denied, but with compensation b. No restriction is imposed on the relationship between b and p_1: if there is denial of service then buyer A loses if $b < p_1$, otherwise the buyer gains.

If buyer A is informed (i.e., knows v_A) and purchases for p_1 in period 1, then buyer A earns utility

$$\mathcal{I}_1 = (v_A - p_1)\, p_o + (b - p_1)(1 - p_o) = v_A p_o + b(1 - p_o) - p_1$$

The informed buyer A purchases if $\mathcal{I}_1 \geq 0$ (because the only other option, "no purchase", yields zero utility). Hence, there exists a unique threshold, $\tilde{v}_A$, such that the informed buyer A purchases in period 1 when $v_A \geq \tilde{v}_A$, where

$$\tilde{v}_A = \frac{p_1 - b(1 - p_o)}{p_o}$$

The uninformed buyer A earns utility

$$\mathcal{U}_1 = \frac{1}{2} p_o + b(1 - p_o) - p_1$$

from a period 1 purchase while waiting to period 2 to attempt to make a purchase earns utility $\mathcal{U}_2 = p_2 (1 - p_2)^2 / 2$. The uninformed buyer A purchases in period 1 if $\mathcal{U}_1 \geq \mathcal{U}_2$, otherwise the buyer waits in period 1 with the hope that maybe a purchase can be made in period 2.

If the firm follows an the advance selling strategy p_1 is chosen so that the uniformed buyer A purchases in period 1. The firm's revenue is

$$\Pi_1 = \beta \tilde{v}_A (1 - p_2)\, p_2 + (1 - \beta \tilde{v}_A)\left(p_1 p_o + (p_1 - b + p_o)(1 - p_o)\right)$$

The first term is revenue when the firm fails to sell in period 1 and the second is revenue when a period 1 sale occurs: with probability p_o buyer B is not willing to pay the high period 2 price, otherwise buyer B pays the higher price and the firm gives compensation b to buyer A because of the denial of service.

Alternatively, if the firm follows a price skimming strategy, the first period price is selected so that the uninformed buyer A does not purchase in period 1. The only buyer who purchases in period 1 is the informed buyer A with a sufficiently high value, i.e., $v_A \geq \tilde{v}_A$. As in the price skimming equilibrium with reselling, equation (1) provides ω, the conditional probability that buyer A participates in period 2, and equation (2) provides Π_2, the period 2 revenue (conditional there is no sale in period 1). Period 1 revenue is

$$\Pi_1 = \beta(1 - \tilde{v}_A)\left(p_1 p_o + (p_1 - b + p_o)(1 - p_o)\right) + \left(1 - \beta(1 - \tilde{v}_A)\right)\Pi_2$$

The first term is revenue when a sale occurs in period 1, including the possibility of a transfer to buyer B in period 2. The second term is revenue when the firm fails to make a sale in period 1.

Theorem 5. *With the overbooking mechanism, there exists a unique threshold β^b,such that*

1. *If $\beta \leq \beta^b$, the firm implements an advance selling strategy. Equilibrium prices are*

$$p_1^* = \frac{3}{16} + \frac{3b^* - 2b^{*2}}{4}, \quad p_2^* = \frac{1}{2}, \quad p_o^* = \frac{1+b^*}{2}$$

and the overbooking payment, $p_1^* < b^* < 1$, is the solution to $32 - 15\beta = 8\beta b^* + (96 - 44\beta)\, b^{*2} + 32\,(2-\beta)\, b^{*3}$. Surplus is

$$S = (1/8)\left(5 + 2b^*\beta\,(1 - v_A^*)\, v_A^* - 2\beta v_A^{*2} - (1 - \beta v_A^*)\, b^{*2}\right),$$

where $v_A^* = \left(2p_1^* - b^* + b^{*2}\right) / (1 + b^*)$.

2. *Otherwise, the firm implements a price skimming strategy. In this equilibrium, $p_o^* = (1 + b^*)/2$, $b^* = p_1^* = \tilde{v}_A^*$,*

$$p_2^* = \frac{-1 + \omega + \sqrt{1 + \omega + \omega^2}}{3\omega}$$

and

$$\tilde{v}_A^* = \frac{1}{3}\left(-1 + 2\sqrt{1 + 3p_2^*\,(1 - p_2^*)}\right),$$

where ω is given by (1). Revenue and surplus in this equilibrium are equivalent to the revenue and surplus in the price skimming equilibrium with the reselling mechanism.

According to Theorem 5, the level of compensation the firm provides depends on its pricing strategy. With advance selling, the firm promises to compensate the buyer for more than the buyer paid in case service is denied, i.e., $b^* > p_1^*$. Even with the possibility for overcompensation, buyer A remains indifferent between purchasing in period 1 or waiting–i.e., buyer A does not stand to gain from this in expectation. However, because buyer A's anticipation for the potential reward, the firm charges a higher price and is better off.

However, with price skimming it is optimal for the firm to merely refund buyer A the original purchase price if buyer A is denied service. Doing otherwise (i.e., $p_1 \neq b$) would introduce costly frictions that do not add value in net. For example, if $p_1 < b$, then buyer A may purchase the unit on speculation and earn a net loss (i.e., $v_A < p_1$ is possible). To compensate buyer A for this possibility, the firm needs to increase the offer price to buyer B, but raising p_o may prevent a profitable trade, and this tradeoff does not work in the firm's favor.

A striking result is that when the firm implements price skimming, the reselling and overbooking mechanisms are equivalent in expectation even though they are not equivalent in implementation - with reselling

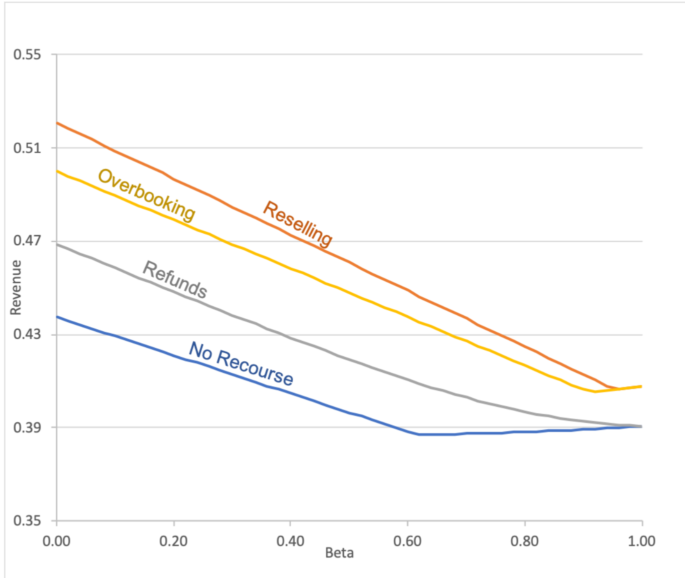


Figure 2. Revenue comparison across the six selling mechanisms

the buyer posts a price conditional on the valuation v_A whereas with overbooking the firm posts a fixed price. In both cases the firm is able to earn the entire surplus from the "transfer market" between the buyers, but it does so in different ways. With reselling the marginal buyer pays the firm a purchase premium, $p_1^* - \tilde{v}_A^*$, which equals the expected gain from the transfer market. With overbooking the firm earns the entire expected gain from the transfer market because $b^* = \tilde{v}_A^*$. Furthermore, each mechanism yields the same value for the marginal buyer, $\tilde{v}_A^*$, which means that each has the same expected gain from the transfer market. In effect, with reselling the firm sells the rights to the transfer market to the buyer, whereas with overbooking the firm keeps those rights for itself.

8 Mechanism comparison

This section compares the different versions of the recourse mechanisms considered in §§4-7. The comparison highlights four results: advance selling is generally preferred to price skimming, but not always; including a recourse mechanism can substantially increase the firm's profit, especially if combined with advance selling; there is considerable variation across recourse mechanisms, with reselling providing the firm with the highest profit as established; and recourse mechanisms are also good for consumers.

Figure 2 graphs for each mechanism the firm's revenue as a function of β (the probability buyer A is informed). When a recourse strategy cannot be implemented the best approach for the firm between advance selling and price skimming depends critically on β, the likelihood that buyer A is informed. When $\beta < 0.615$, advance selling is preferred, otherwise price skimming is better. And the differences between these strategies can be substantial. For example, when buyer A is surely uniformed ($\beta = 0$, which is typically assumed in the literature), revenue with advance selling is 13.67% higher than with price skimming. Advance selling performs well with a low β because the firm can set a sufficiently low price to induce the buyer to pay the expected value for the unit. This maximizes the probability of a transaction, which benefits the firm despite the lower price. At the other extreme, when buyer A is surely informed, i.e., $\beta = 1$, the firm's revenue with advance selling is 9% lower than with price skimming. In situations with a high β advance selling is no longer guaranteed to make the transaction (even the lower price might not be low enough for an informed buyer with a low value). It is better to set a declining price path (i.e., price skimming) to try to identify the buyer with the highest value (the other source of value creation).

Adding a recourse mechanism generally increases the firm's revenue substantially no matter the likelihood that buyer A is informed of the value v_A or not. For example, the best recourse mechanism (reselling) yields between 4.3% (when β is high) and 19.0% (when β is low) higher revenue than the best non-recourse mechanism (either may be done with advance selling or price skimming, depending on β). Recourse mechanisms are good for the firm because they create additional value (increase the likelihood of a transaction and/or increase the likelihood the buyer with the highest value acquires the unit) and the firm captures some of that additional value. The one case in which recourse does not help is with refunds and price skimming - a refund does not benefit the potential period 1 buyer (who is informed in this equilibrium), so it does not generate additional surplus and it does not increase the firm's revenue.

Recourse mechanism are more useful when combined with advance selling than with price skimming, because recourse mechanisms directly address the limitation of advance selling. To explain, the firm needs to lower its price with advance selling to compensate the buyer for the uncertainty associated with v_A. One concern is that the buyer's value may be relatively low. Refunds provide some "insurance" against that bad outcome by placing a floor on the utility the buyer can receive, i.e., $\max\{f, v_A\}$. A second concern is that a buyer might not have the highest value. Reselling and overbooking help to mitigate that issue - a buyer can benefit if the other buyer is willing to pay a high price (because of a higher value). In contrast, with price skimming, buyer A in period 1 knows v_A, so does not need the insurance against a low value. And because the period 1 price is relatively high, it is less likely that the buyer can benefit from a transfer to the other buyer - the buyer purchases only if the value is relatively high to start, leaving less upside to gain from a transfer.

Not all recourse mechanisms are the same. In the case of advance selling, reselling is clearly the best, followed by overbooking, followed by refunds. Reselling and overbooking are better than refunds because they focus on transferring the unit to the buyer with the highest value, which is more important than providing protection for a buyer against a low value (as is done with refunds). Reselling is better than overbooking

β	Π_0^*/S_0^*	Π_r^*/S_r^*	Π_f^*/S_f^*	Π_b^*/S_b^*
0	87.5%	83.3%	83.3%	84.2%
0.2	82.0%	80.4%	79.4%	80.7%
0.4	76.7%	77.4%	75.7%	77.2%
0.6	71.8%	74.3%	72.7%	73.2%
0.8	72.8%	71.2%	71.5%	69.7%
1	72.5%	72.0%	72.5%	72.0%

Table 1. Fraction of surplus captured by the firm. Π_i^* and S_i^* are the firm's optimal revenue and the resulting social welfare under the four mechanisms: subscript 0 for no recourse, r for resale, f for refund and b for overbooking. β is the probability buyer A is informed of v_A in period 1.

with advance selling because of who sets the transfer price. With reselling it is the buyer that determines the resale price whereas with overbooking it is the seller that determines the price. Because the buyer knows v_A, the resale price can accurately reflect the buyer's value. In contrast, with overbooking the firm sets a transfer price without knowing either buyer A's value or buyer B's value. It is entirely possible that buyer A has the higher value, yet the firm removes the unit from A and gives it to B. That mistake, which reduces the expected surplus in the system, never occurs with reselling. While it may seem counterintuitive that the firm prefers to relinquish control over its capacity, it in fact prefers to let the buyer set the resale price, because the buyer has value for the unit (and knows it) whereas the firm does not. The firm benefits because it is able to capture some of this value through its initial transaction with the buyer, i.e. by charging a high price in period 1.

Finally, we find that all recourse mechanisms increase the system's surplus. Surplus does not depend on the transfers between the agents. It does depend on the probability of a transfer and on the chance the buyer with the highest value receives the unit. Recourse mechanisms improve upon both. But does the increase in system surplus "lift all boats", i.e., are consumers also better off? That depends on how the recourse mechanism changes the portion of the surplus generated the firm is able to snatch for itself. If that fraction is relatively stable, then consumers are better off (i.e., they get the same fraction of a larger pie). But if the recourse method allows the firm to capture a significantly larger portion of the generated surplus, then consumers may actually be harmed despite the additional surplus available in the system. Table 1 reports on the share of surplus captured by the firm across different recourse mechanisms. For a fixed β the firm's share when a recourse mechanism is added either decreases (which surely benefits consumers) or increases only slightly, suggesting that consumers too benefit from recourse mechanisms. Table 2 confirms that consumers always benefit from recourse mechanisms, with the largest percentage increases occurring when they are likely to be uniformed (with low β) .

β	C_r^*/C_0^*	C_f^*/C_0^*	C_b^*/C_0^*
0	167%	150%	150%
0.2	131%	126%	124%
0.4	113%	112%	111%
0.6	101%	101%	104%
0.8	118%	109%	124%
1	107%	100%	107%

Table 2. Percent change in consumer surplus when a recourse mechanism is adopted. C_i^* is consumer surplus with mechanism i: 0 for no recourse, r for resale, f for refund and b for overbooking. β is the probability buyer A is informed of v_A in period 1

9 Discussion

A firm selling perishable capacity over time needs to create value and then extract some of that value. Value is created in two ways: (i) transfer the capacity to a buyer (the firm has zero value for the capacity at the end of the horizon) and (ii) conditional on a transfer, ensure the capacity is used by the buyer with the highest value. This framework can be used to understand the relative performance of different mechanisms. Advance selling is better than price skimming at ensuring the capacity is transferred to some buyer, but it is less effective at placing the capacity with the buyer with the highest value. Recourse mechanisms focus on proper allocation, but they differ in their approach: reselling puts the buyer in charge of transferring, while with refunds both agents are involved (the buyer needs to return the unit and the firm then needs to find a suitable other buyer), and overbooking keeps the firm in control of the transfer. Given that recourse is centered on "finding the buyer with the highest value", and this is the main limitation of advance selling, recourse strategies tend to improve revenue more with advance selling than with price skimming.

Regardless of whether the firm implements advance selling or price skimming, resale is the best recourse mechanism. In fact, not only is reselling the best recourse mechanism among the ones we consider, Theorem 2 establishes that it is optimal among all mechanisms. Consequently, there is no need to combine recourse mechanisms: e.g., the pairing of reselling with refunds or overbooking cannot do better than reselling alone. But if reselling is optimal, why don't firms always allow resale in practice? Several factors that we do not model directly may diminish the advantage of resale:

Timing: In our model timing is not constraining so that a potential seller can always be matched with a potential buyer to enable the possibility of a trade. That is, both the firm and buyer A are always able to find buyer B for a resale market to occur. If time is limited so that locating a potential buyer is not guaranteed, it puts the transaction at risk and the benefit from recourse strategies reduces. Conceptually, it is likely that firms have the capabilities to locate new buyers quicker than other buyers can. This implies that time constraints may harm resale strategies more than they do overbooking and refund strategies which reduces the benefit of resale.

Transaction costs: Throughout we have considered zero transaction costs. Reselling requires consumer interaction and an ability for sellers to "find" potential buyers. To facilitate these matchings, firms often set up resale websites (this is commonly done by sport teams–all four major leagues now have sponsored resale marketplaces) or allow the use of third party platforms (e.g., Stubhub and Ticketmaster), all of which involve various costs. Refunds require some communication between consumers and the firm. Overbooking does not involve matching costs, and the communication costs are minimal, but it does impose non-trivial psychological costs - even though buyer A might anticipate being denied service, in practice it is possible that this event imposes additional disutility beyond what we model. Naturally, any of these transaction costs introduce market inefficiencies that reduce the value of a recourse mechanism. To isolate our results from such inefficiencies, we purposefully assume zero transaction costs so that we can rank recourse mechanisms. However, the final ranking of these mechanisms could change depending on the differences in transaction costs. For example, if actual transaction costs with overbooking are substantially lower than with reselling, overbooking could be preferred. And if transaction costs across all recourse mechanisms are too high, the firm might prefer the no-recourse mechanism.

Competition in the resale market: In our model there is one unit of capacity and one buyer in the first period, so there is no competition in the resale market: either the firm or buyer A own the unit in period 2. What if capacity was less limiting? Consider first the extreme case in which the firm has two units available to sell in period 1 instead of just one. As there are two buyers and two units to sell, capacity is not restrictive. With no recourse, it is now even more important to ensure a transfer than it is to ensure the proper allocation because every unit of demand can be satisfied. Consequently, advance selling is superior to price skimming for a larger set of β: with two units to sell the firm prefers advance selling for $\beta < 8/9$ whereas, as reported earlier, with one unit to sell advance selling is preferred for $\beta < 0.615$. Given that proper allocation is not important, when capacity is ample, the benefit of recourse diminishes entirely to zero. Say the firm could offer reselling. If the firm sells a unit in period 1, it competes over the sale of its remaining unit with Buyer A in period 2. Because the firm has zero value for the unit, but Buyer A's value is positive, the firm surely posts the lower price in period 2 (it can undercut any price that the buyer is posting) and consequently the firm is the only agent that can make a sale in period 2. In that case, a resale mechanism is equivalent to a no recourse mechanism – Buyer A gains nothing from the possibility to resell and the firm cannot profit from allowing resale. If capacity is not restrictive, the same is true for all other recourse mechanisms as well. With two units to sell and two buyers, the firm does not benefit from offering refunds or from overbooking either. The story is somewhat different if capacity can create competition in the resale market, but is still potentially restrictive. Then the effectiveness of resale decreases, because competition in the resale market introduces a tradeoff for the firm. On one hand, the firm can charge a high price in period 1, because buyer A expects a potential gain from resale. On the other hand, a firm that sold one unit faces competition on the sale of the second unit in the resale market (without resale it is a monopoly). It is possible that this pressure on the resale price reduces the benefit of the resale mechanism. A firm that practices other recourse strategies that do not involve buyers reselling does not face this pressure. That suggests that overbooking, which performs well relative to resale without competition (it is equivalent to reselling when β is high and achieves 96% of the optimal in the worst case), becomes an attractive mechanism to consider, especially if there is a risk of resale market competition.

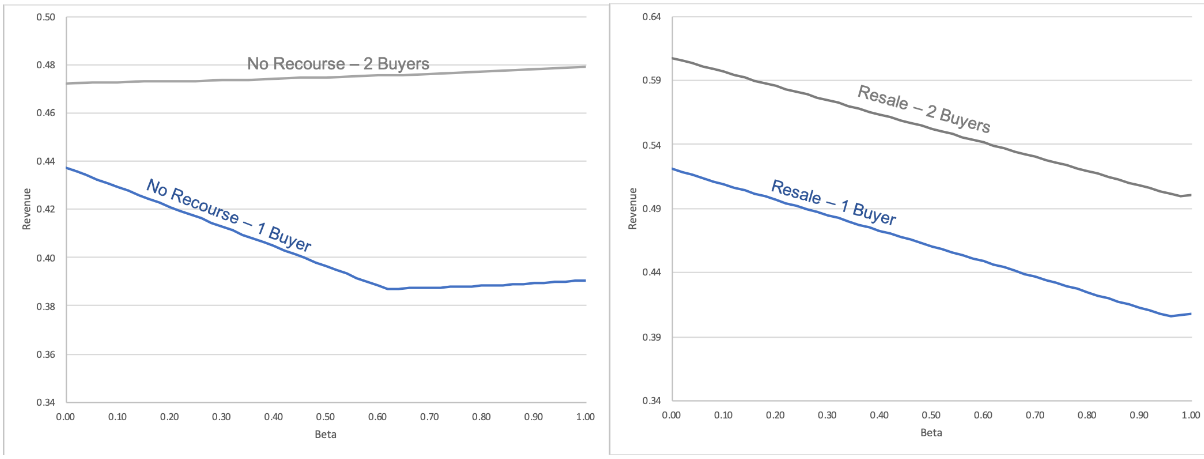
Even when capacity is not constraining the benefit of resale can be restored if the firm could commit to not participate in the resale market. There is still some cost to the firm from having buyer A as an additional seller, because it faces an increased risk of not being able to sell its entire capacity, but the gain from the increased period 1 price may more than outweigh that cost. Anecdotally, in markets which allow resale we observe that firms do not respond with lower prices. Furthermore, while we show that the firm sets zero transaction fees when it does not face competition in the resale market, the same need not be true with competition. Transaction fees can serve as a means to restrict competition with buyers and may indeed be optimal in these situations.

The flip side of additional capacity is additional demand. If capacity were more constraining (i.e., there is more demand), then we would expect that the balance between ensuring a transfer and proper allocation would tip towards proper allocation - when demand is ample there is a small risk that capacity goes unused and there is more to gain from finding the buyer with the highest value. Thus, increasing demand should favor price skimming over advance selling when there is no recourse, which is the case when a second buyer B enters the market, as reported in the first part of Theorem 6. When recourse is feasible, there is less of a tension between ensuring a transfer and proper allocation because the recourse mechanism is able to help correct any allocation error. Hence, with recourse the additional demand should have less of an impact on the relative benefit of advance selling and price skimming. The second part of Theorem 6 and Figure 3 demonstrate this is the case when a second buyer B is added to the market and reselling is the recourse mechanism.

Theorem 6. *With two buyer Bs, the following equilibria hold:*

1. *With the no-recourse mechanism the firm implements a price skimming strategy for all β. The firm's revenue function is increasing in β and is higher than when there is a single buyer B.*

2. *With the reselling mechanism, there exists a unique β^r, such that if $\beta \leq \beta^r$, the firm implements an advance selling strategy and otherwise, the firm implements a price skimming strategy. Furthermore,*



(a) Revenue comparison with no recourse **(b)** Revenue comparison with a resale mechanism

Figure 3. Revenue comparison with two Buyers B

the revenue with advance selling decreases in β and the revenue with price skimming increases in β.

In sum, when capacity is tight (i.e., ample demand), the firm should use price skimming if a recourse mechanism is not feasible, but if recourse is feasible, then advance selling is preferred. In contrast, when capacity is ample (i.e., demand is limited), advance selling is the preferred mechanism and recourse is not (as) useful.

Our model presumes that the firm not only has the opportunity to sell its capacity over time, it chooses to do so. It is straightforward to confirm that this is the correct choice for the firm. If the firm were to sell in one period, it would do so in the second period. It could use a posted price mechanism or, if feasible, the revenue maximizing mechanism, which (as already discussed) is a second price auction with reservation price 1/2 (Myerson 1981). Neither of those approaches, according to Figure 4 does well - when consumers arrive sequentially and especially when they have a preference for resolving ownership in advance then the firm should indeed sell its capacity over time.

We consider a model in which the firm sells a single good, e.g., one stay at a hotel, one cabin on a cruise or one itinerary on an airline. However, there are markets in which consumers might have value for multiple products. For example, a consumer could have an interest to attend several sporting events from their home team, a consumer might wish to visit a gym on multiple occasions or go to the movie theater several times a month. It has been shown that the firm can benefit from advance selling of a bundle of multiple goods (e.g. Brynjolfsson 1999, Cachon & Feldman 2011) and, in fact, several startups are offering subscription services in industries that until recently operated based on a price per attendance model (e.g., Sinemia and MoviePass for movie theater subscriptions and Skyhi for airline tickets). If capacity isn't particularly constraining (e.g., gym attendance), the emphasis should be on ensuring a transaction rather than allocating the capacity to

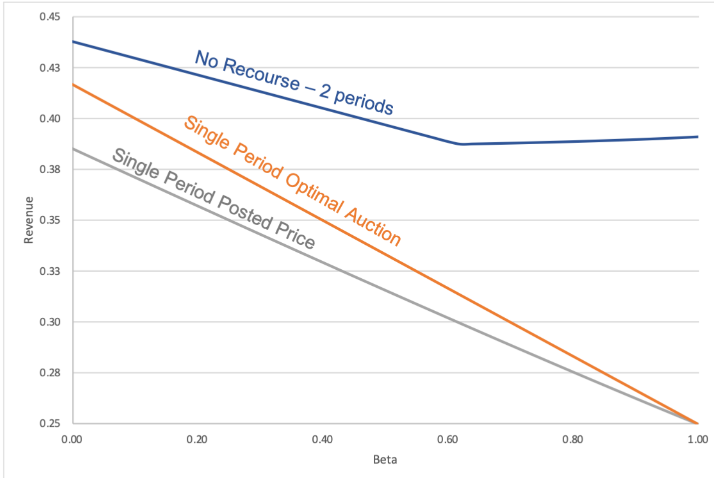


Figure 4. Revenue comparison when selling in a single period

the consumer who values it the most. For example, on any particular day consumers that value exercise actually go the gym and those that don't want to work out stay home. Reselling is unlikely to add much value in those situation, especially if transaction costs are high relative to the value of the item (such as gym attendance on one day). In contrast, if the bundle of goods is attendance at a sporting event, it is important to allocate seats/tickets to the consumers most willing to pay, suggesting that reselling in those markets, even with bundled selling, is desirable. In fact, in a study of MLB ticket sales, Sweeting (2012) finds that 88% of the sellers on eBay sold a single ticket suggesting that many sellers are season ticket holders who are unwilling or unable to attend all the games in the bundle they purchased.

There has been significant debate on the practice of reselling. A major concern is the presence of speculators who purchase capacity in advance with the hope of later selling the capacity at a higher price. Such behavior is viewed at best as a loss of revenue to the firm, and at worst as a violation of ethical norms. However, we argue that speculators can only enter a market that faces clear inefficiencies, such as rigid prices. The presence of speculators should be viewed as a symptom of a market failure because they provide no intrinsic value to the system - they neither create the capacity nor the value from its use. In our model, due to the absence of market frictions, there is no value for speculators and they are unable to profitably enter. Instead, reselling is done by the consumers themselves, as is currently observed in many markets. Unlike the firm, the consumer knows the value for the capacity and therefore is better able to set a resale price. Nevertheless, the firm can anticipate this value and price accordingly with the initial transaction. In effect, the firm is able to extract from the marginal consumer the entire additional value the consumer could earn from product resale, thereby benefiting from resale as well.

10 Conclusion

We study how a firm should price it's limited perishable capacity over time. The firm can try to sell to consumers before they know their value for the good with certainty (advance selling) or it can try to screen customers to find the one with the highest value (price skimming). After a transaction is made circumstances can change, motivating the firm or the customer to desire a change in the initial terns, i.e., a recourse. For example, a customer may wish to try to resell (if allowed) the good to another customer, or the customer could return the unit to the firm for a partial refund, or the firm could try to find a buyer willing to pay even more than the first buyer, a practice called overbooking. Previous research has considered some of these mechanisms in isolation, but we present a unified framework that can be applied to understand each strategy and its relative performance for the firm and consumers.

Despite previous research that extols the virtues of advance selling, we identify a significant limitation - a firm cannot know for sure when customers learn their true value for a good. Without that knowledge, the firm cannot perfectly time its pricing changes to that event. Consequently, while advance selling can be better than price skimming, it isn't always better.

Among the recourse strategies, it would appear that reselling and refunds favor consumers (because they initiate the recourse) while overbooking might favor the firm. However, all are beneficial to the firm, with reselling generally the best, followed by overbooking and then refunds. Furthermore, recourse strategies generally are more beneficial when combined with advance selling than with price skimming, because recourse strategies address the main weakness of advance selling - while advance selling increases the probability of a transaction (which generates value for the system), it does little on its own to allocate the unit to the buyer with the highest value, and all recourse strategies focus on improving the odds the consumer with the highest value is assigned the unit. Interestingly, even though the firm designs the terms of the recourse strategy, consumers also benefit from the implementation of a recourse strategy - adding a recourse mechanism increases the total value in the system without radically shifting the share of that value allocated between the firm and consumers.

Recourse strategies are very effective in our model in part because we have ignored the non-zero transaction costs associated with actually implementing recourse. In markets with high transaction costs, recourse strategies might not be desirable. However, the use of information technology generally reduces transaction costs, making these recourse strategies potentially feasible and even highly profitable. It follows that it is in the firm's interest to reduce transaction costs as much as possible to encourage the use of recourse strategies.

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A Proofs

Proof of Theorem 1. Advance Selling: The firm's revenue function is increasing in p_1 if $\beta \leq 4/7$ and concave if $\beta > 4/7$. Thus, if $\beta \leq 4/7$, the firm chooses the maximum period 1 price that satisfies the $\mathcal{U}_2 \leq \mathcal{U}_1$ constraint. Otherwise, because there is a one-to-one correspondence between p_1 and $\tilde{v}_A$, the firm's problem can be written in terms of $\tilde{v}_A$:

$$\Pi_1 = \frac{1}{4}\left((1-\beta\tilde{v}_A)(1+\tilde{v}_A)^2 + \beta\tilde{v}_A \right),$$

which is concave in $\tilde{v}_A$ and maximized at

$$\tilde{v}_A = \frac{1-2\beta+\sqrt{1+2\beta+4\beta^2}}{3\beta}.$$

To satisfy the $\mathcal{U}_1 \geq \mathcal{U}_2$, we must have that $\tilde{v}_A \leq 5/\left(2\sqrt{3}\right) - 1$, which is always violated in this range of β. Therefore, the firm chooses the maximum period 1 price that satisfies the constraint for all β.
Conditional that buyer A has value v_A and has the unit at the start of period 2, expected surplus is

$$S\left(v_A\right) = \frac{1}{2}\left(1 - p_r^2\right) + p_r v_A = \frac{1}{8}\left(3 + 2v_A + 3v_A^2\right).$$

The first terms is the Buyer B surplus from reselling the unit to him and the second term is Buyer A's value conditional on keeping the unit. Let $S_{\tilde{v}_A}$ be the expected surplus from buyers with values $[\tilde{v}_A, 1]$

$$S_{\tilde{v}_A} = \int_{\tilde{v}_A}^{1} S\left(x\right) dx = \frac{1}{8}\left(5 - 3\tilde{v}_A - \tilde{v}_A^2 - \tilde{v}_A^3\right)$$

Overall surplus is

$$S = \left(1 - \beta\right) S_0 + \beta\left(S_{\tilde{v}_A} + \frac{1}{2}\tilde{v}_A\left(1 - p_2^{*2}\right)\right),$$

where

$$S_0 = \int_0^1 \frac{1}{8}\left(3 + 2v + 3v^2\right) dv = \frac{5}{8}$$

is the expected ex-ante surplus earned by an informed buyer A.

Price Skimming: The period 2 revenue function is strictly concave in p_2 and maximized with

$$p_2^* = \frac{-1 + \omega + \sqrt{1 + \omega + \omega^2}}{3\omega}$$

where ω is given by (1). Given $\omega \in [0, 1]$, it follows that $p_2^* \in [1/2, 1/\sqrt{3}]$.

Because there is a one-to-one relationship between $\tilde{v}_A$ and p_1, the equilibrium can be defined in terms of $\tilde{v}_A$ instead of p_1. The revenue function is

$$\Pi_1 = \tfrac{1}{4}\beta(1 - \tilde{v}_A)(1 + \tilde{v}_A)^2 + (1 - \beta + \beta\tilde{v}_A)\,\Pi_2$$

Differentiate the period 1 revenue function

$$\frac{d\Pi_1}{d\tilde{v}_A} = \frac{\partial\Pi_1}{\partial\tilde{v}_A} + (1 - \beta + \beta\tilde{v}_A)\left(\frac{\partial\Pi_2}{\partial p_2^*}\frac{\partial p_2^*}{\partial\tilde{v}_A} + \frac{\partial\Pi_2}{\partial\omega}\frac{\partial\omega}{\partial\tilde{v}_A}\right)$$

Given that $\partial\Pi_2/\partial p_2^* = 0$,

$$\begin{aligned}\tfrac{d\Pi_1}{d\tilde{v}_A} &= \tfrac{1}{4}\beta\left(1 - 2\tilde{v}_A - 3\tilde{v}_A^2\right) + \beta\Pi_2 + (1 - \beta + \beta\tilde{v}_A)\tfrac{\partial\Pi_2}{\partial\omega}\tfrac{\partial\omega}{\partial\tilde{v}_A} \\ &= \beta\left[\tfrac{1}{4}\left(1 - 2\tilde{v}_A - 3\tilde{v}_A^2\right) + p_2^*\left(1 - p_2^*\right)\right]\end{aligned}$$

Solving the first-order condition (FOC), $d\Pi_1/dp_1 = 0$, yields

$$\tilde{v}_A^* = \frac{1}{3}\left(-1 + 2\sqrt{1 + 3p_2^*\left(1 - p_2^*\right)}\right)$$

To demonstrate that the FOC yields a unique and maximum solution, it is sufficient to demonstrate that for all solutions to the FOC $d\Pi_1^2/d\tilde{v}_A^2 \leq 0$.

$$\frac{d^2\Pi_1}{d\tilde{v}_A^2} = \beta\left[-\frac{1}{2} - \frac{2}{3}\tilde{v}_A + (2p_2^* - 1)\frac{dp_2^*}{d\omega}\left(-\frac{\partial\omega}{\partial\tilde{v}_A}\right)\right] < 0$$

The sign above follows because $p_2^* \leq 1/\sqrt{3}$, $\partial p_2^*/\partial\omega \leq 3/24$, $1/3 < \tilde{v}_A$ (from the FOC), and $-\partial\omega/\tilde{v}_A \leq 3/4$. It remains to confirm that the uniformed buyer A indeed does not purchase in period 1. Not purchasing in period 1 yields the uninformed buyer A an expected utility of $p_2^*(1-p_2^*)^2/2$. Purchasing in period 1 yields expected utility $7/12 - p_1^*$. The uninformed buyer A follows the equilibrium path if

$$\frac{1}{2}\left(\frac{7}{6} - p_2^*(1-p_2^*)^2\right) \leq p_1^*$$

which holds given the derived bounds on the prices.
The revenue function is increasing in β :

$$\frac{d\Pi_1}{d\beta} = \frac{\partial\Pi_1}{\partial\beta} + \frac{\partial\Pi_1}{\partial p_1}\frac{\partial p_1}{\partial\beta} = (1-\tilde{v}_A^*)(p_1^* - \Pi_2) > 0$$

where the inequality follows from the derived bounds on the prices.
Surplus is

$$S^* = \beta\left[S_{\tilde{v}_A} + \tilde{v}_A(1-p_2^*)\left(\frac{1+p_2^*}{2}\right)\right] + (1-\beta)\left((1-p_2^*)\left(\frac{1+p_2^*}{2}\right) + p_2^*(1-p_2^*)\left(\frac{1+p_2^*}{2}\right)\right):$$

An informed customer yields $S_{\tilde{v}_A}$ if he buys and if he does not buy (with probability $\tilde{v}_A$) only buyer B remains on the spot yielding an expected surplus of $(1+p_2^*)/2$ if he buys. An uninformed customer waits. The expected resulting surplus is buyer B's expected value if he buys $(1+p_2^*)/2$ or, if he does not, buyer A's may purchase.

□

Proof of Theorem 2. According to the revelation principle (Myerson 1979) any equilibrium of an indirect (i.e., possibly non-truth inducing) optimal mechanism can be implemented as a truth-inducing direct mechanism. In the optimal direct mechanism in period 1 buyer A announces his type, which can be some value, "v_A", or "unsure". Given this report, the firm either transfers the unit to A for a payment, which can depend on the report, or does not transfer the unit. Per the definition of a direct mechanism, in period 1 buyer A must have an incentive to truthfully report his type.

If there is a transfer of the unit in period 1 to buyer A, then it must occur at a fixed price no matter buyer A's report: if buyer A's period 1 report influenced the transfer price, then buyer A would have an incentive to report the type that leads to the lower price, even if that isn't buyer A's type. Hence, there are two options for the direct mechanism. In both options buyer A receives the unit in period 1 for price p_1 if the buyer reports a sufficiently high "v_A". Let that threshold value be $\overline{v}_A$.

In the first option for a direct mechanism (price skimming), a transfer occurs only when buyer A reports "high v_A" (i.e., a v_A value such that $\overline{v}_A \leq v_A$). The firm retains the unit if the buyer reports "unsure" or "low v_A". In the second option for a direct mechanism (advance selling), the firm transfers the unit to buyer A in period 1 for p_1 if the buyer reports a "high v_A" or "unsure". Again, the firm retains the unit with a "low v_A" report.

In period 2, either buyer A has the unit or the firm has the unit. If buyer A has the unit, then the buyer can attempt to transfer the unit to buyer B, possibly with restrictions imposed by the firm. If the firm has the unit, then the period 2 market may have one or two potential buyers. The firm can sell to these buyers, possibly restricted by commitments it made in the mechanism.

With a direct mechanism, in period 1 the firm learns information that reveals the number of buyers in period 2 - in equilibrium if the report is "low v_A" then there is only buyer B in period 2, but if the report is "unsure" and the firm keeps the unit, then both buyers are *present* in period 2. Although the firm obtains this information, the firm cannot use it in a way that would disadvantage buyer A. If it were to use the information against buyer A, then buyer A would have an incentive to misreport the buyer's type. Hence, the firm must implement a mechanism in period 2 that yields buyer A the same expected value no matter if the unsure buyer A reports "unsure" (which, with the second option, implies there are two buyers in period 2) or "low v_A" (which implies there is only one buyer in period 2).

Period 2: In period 2 the unit is either owned by the firm or buyer A. If the firm has the unit, then the firm can implement any period 2 mechanism so long as the truth-inducing constraint in period 1 is not violated. If buyer A owns the unit in period 2, then the buyer may attempt to sell the unit to B, subject to the firm's restrictions. For example, the firm could charge a commission on the sale (or charge a fixed fee, etc.).

Suppose that the firm sells in period 2. Consider the revenue maximizing mechanism and a posted price mechanism:

Revenue maximizing mechanism: In period 2 the firm maximizes that period's revenue with a second-price auction with a reserve of $p_2 = 1/2$. The form of the mechanism does not depend on the number of buyers in period 2. Therefore, it satisfies the truth-inducing requirement in period 1 that the period 2 mechanism does not depend on the firm's expectations for the number of buyers present in period 2. The expectations for the firm's revenue, r_2, and a buyer's utility, π, depends on how many buyers are in the market:

	1 buyer	2 buyers	2 buyers with probability ω
r_2	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{4}+\frac{1}{6}\omega$
π	$\frac{1}{8}$	$\frac{1}{12}$	

Posted price: A posted price mechanism with one buyer achieves the same revenue as the second-price auction. With two or more buyers, the posted price achieves less revenue. Nevertheless, it can be a useful mechanism for the firm (as explained in the period 1 analysis). There are two cases to consider: (i) only buyer B, and (ii) there is one buyer with probability $1-\omega$ and two buyers with probability ω. With a single buyer B:

price	firm revenue, r_2	π_A: informed buyer A	π_A: unsure buyer A
1/2	1/4	0	$\pi = 1/16 = 0.0625$

With a ω probability of two buyers the firm's revenue is

$$r_2 = \omega\left(1-p_2^2\right)p_2 + (1-\omega)(1-p_2)\,p_2$$

The optimal period 2 price is

$$p_2^* = \frac{-1+\omega+\sqrt{1+\omega+\omega^2}}{3\omega}$$

The unsure buyer A earns $\pi = p_2^*\left(1-p_2^*\right)^2/2$.

Suppose next that buyer A is selling in period 2. In period 2 buyer A is always informed. The worst a buyer A can do is merely keep the unit and receive $\pi = v_A$. However, buyer A could do better if there is an opportunity to sell to buyer B. Suppose for any transaction with buyer B the firm takes a portion ϕ of the transfer price and charges a fee f. Buyer A's final utility is $u = \pi - c$, where π is the buyer's utility before commissions, $\pi = p_2 v_A + (1-p_2)\,p_2$ and c is the expected commission to the firm, $c = (1-p_2)(\phi p_2 + f)$. Buyer A's best price is

$$p_2 = \begin{cases} \frac{1}{2}\left(\frac{1-\phi+v_A+f}{1-\phi}\right) & \phi < 1-v_A-f \\ 1 & 1-v_A-f < \phi \end{cases}$$

The resulting final utility is

$$u = \begin{cases} \frac{1}{4}\left(\frac{(1-\phi+v_A)^2-2f(1-\phi+v_A)+f^2}{1-\phi}\right) & \phi < 1-v_A-f \\ v_A & 1-v_A-f < \phi \end{cases}$$

The expected commission (assuming $\phi < 1-v_A-f$) is

$$c = \frac{(1-\phi-v_A-f)\left(\phi\left(1-\phi+v_A\right)+f\left(2-\phi\right)\right)}{4\left(1-\phi\right)^2}$$

Prove that the firm's optimal commission is to not charge a commission, i.e., $\phi = 0$ and $f = 0$. Proof by contradiction. Say the optimal commission contract has either $\phi > 0$ or $f > 0$ (or both) for some subset of buyer A types. For that contract let p_1 be the period 1 price, and define u, π,and c as above. Let $\underline{v}_A$ be the lowest value of the informed buyer As that is willing to purchase the contract, i.e., $p_1 \leq u$. Let p_2 be the period 2 price that buyer posts to sell the unit. Let u^0, π^0, and p_2^0 be the analogous values for the zero commission contract. For the $\underline{v}_A$ buyer, $p_2^0 < p_2$ and $\pi < \pi^0$: without a commission the buyer charges less to sell the unit and earns a higher utility without consideration of commissions (because p_2^0 maximizes the buyer's utility net of commissions). The firm's revenue from this buyer is $p_1 + c$. Say the maximum price is charged with the zero commission contract, i.e., $p_1^0 = u^0$. Because $\pi < \pi^0$, the firm's total earnings from this buyer is larger than the earnings with the commission contract: $p_1^0 = u^0 = \pi^0 > \pi = u + c \geq p_1 + c$. Furthermore, because c is decreasing in v_A, it follows that $p_1^0 > p_1 + c$ for all buyers with values greater than v_A. Given that the zero commission contract earns more for the firm for all buyers, it is the optimal contract. In the remainder assume the zero commission contract is used.

With zero commission buyer A's price is $p_2 = (1 + v_A)/2$ and the buyer's utility is $\pi = (1 + v_A)^2/4$.

Period 1: Consider the first mechanism option (price skimming): an announcement of $\overline{v}_A$ or higher awards the unit to buyer A for a payment of p_1, otherwise the firm retains the unit. If the buyer A announces less than $\overline{v}_A$ or "unsure", then the firm commits to a period 2 mechanism that leaves the unsure buyer A at least as well off reporting "unsure" as "low v_A". The informed buyer A reports his value in period 1 only if doing so earns a non-negative utility, i.e., if $p_1 \leq (1 + v_A)^2/4$. The firm's revenue is $R = \beta(1 - \overline{v}_A)p_1 + (1 - \beta + \beta\overline{v}_A)r_2$, where $r_2 = 1/4 + 1/(6\omega)$ and

$$\omega = \frac{1 - \beta}{1 - \beta + \beta\overline{v}_A}$$

For a fixed $\overline{v}_A$ the firm maximizes p_1. The firm's revenue can then be expressed in terms of $\overline{v}_A$: $R = \left(5 - \beta\left(2 - 6\overline{v}_A + 3\overline{v}_A^2 + 3\overline{v}_A^3\right)\right)/12$. The firm's optimal solution is $\overline{v}_A = \left(\sqrt{7} - 1\right)/3$ and the resulting period 1 price is $p_1 = \left(11 + 4\sqrt{7}\right)/36$ and revenue is

$$R = \frac{1}{108}\left(45 - 2\left(19 - 7\sqrt{7}\right)\beta\right)$$

The period 1 price does not depend on β but revenue decreases linearly in β and revenue is relatively insensitive to β :

$$\frac{R(0)}{R(1)} = \frac{45}{45 - 2(19 - 7)} = 1.011$$

It remains to check that all types truthfully report their type. By construction, the informed buyer A reports truthfully. The unsure buyer A is indifferent between reporting "unsure" and a low v_A because either way the buyer's utility in period 2 is the same, which is 1/12. The unsure A does not want to report "high v_A" to earn the unit in period 1, because then the buyer earns a negative payoff:

$$\int_0^1 \frac{1}{4}(1 + x)^2\,dx - p_1 = \frac{7}{12} - \frac{1}{36}\left(11 + 4\sqrt{7}\right) < 0$$

Consider the second mechanism (advance selling): buyer A that announces no less than $\overline{v}_A$ or "unsure" gets the unit for p_1, and buyer A that announces less than $\overline{v}_A$ does not get the unit and the firm assumes it is selling to a single buyer in period 2 with value $[0, 1]$. The firm's revenue is $R = \beta((1 - \overline{v}_a)p_1 + \overline{v}_a/4) + (1 - \beta)p_1$. If $\overline{v}_a = 2\sqrt{p_1} - 1$ then the informed buyer A earns zero from a false report (because the unit is of no value in period 2). Hence, the revenue function simplifies to

$$R = p_1 + \beta\left(-\frac{1}{4} + \frac{1}{2}\sqrt{p_1} + p_1 - 2p_1^{3/2}\right)$$

The above is increasing for the feasible range of p_1, so the optimal p_1 is maximized subject to the truth-telling constraint for the unsure buyer A. The unsure buyer A that correctly reports "unsure" earns $\pi = 7/12 - p_1$. To maximize p_1, the firm wants to minimize the earnings of buyer A from a false report. This is achieved with a posted price mechanism in period 2 (the second price auction earns the same amount in equilibrium, but yields more for buyer A off the equilibrium path), in which case buyer A earns $\pi = 1/16$. Thus, the firm can charge

$$p_1 = \frac{7}{12} - \frac{1}{16} = \frac{25}{48}$$

The resulting revenue is

$$R = \left(150 - \left(65\sqrt{3} - 78\right)\beta\right)/288.$$

□

Proof of Theorem 3. Advance Selling: The second period revenue function is concave and maximized with price $p_2^* = 1/2$. The period 1 revenue function is concave in p_1. However, the unconstrained optimal price violates the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint. Given that constraint binds, the optimal period 1 price is $p_1^* = 7/16$. Surplus is

$$S^* = \frac{1}{2}\beta\left[1 - p_1^{*2} + p_1^*\left(1 - p_2^{*2}\right)\right] + \frac{1}{2}(1-\beta)$$

With probability β, Buyer A is informed. If $v_A \geq p_1^*$ (with probability $(1-p_1)$), the unit is transferred to him and the expected value is $\mathbb{E}\left[V|V \geq p_1^*\right] = \left(1 + p_1^*\right)/2$. Otherwise, in period 2 the unit may be sold to Buyer 2 and expected value is $p_2^*\mathbb{E}\left[V|V \geq p_2^*\right] = \left(1 - p_2^{*2}\right)/2$.
Price Skimming: In equilibrium the constraint $\mathcal{U}_1 < \mathcal{U}_2$ ensures that the uniformed buyer A does not purchase in period 1:

$$\frac{1}{2}\left(1 - p_2\left(1-p_2\right)^2\right) \leq p_1$$

The period 2 revenue function is concave in p_2, so the optimal p_2 solves the first-order-condition, $d\Pi_2/dp_2 = 0$. Given $\omega \in [0,1]$ it is straightforward to show that $p_2 \in [1/2, 1/\sqrt{3}]$. From the period 1 revenue function

$$\frac{d\Pi_1}{dp_1} = \frac{\partial \Pi_1}{\partial p_1} + (1 - \beta + \beta p_1)\left(\frac{\partial \Pi_2}{\partial p_2}\frac{\partial p_2}{\partial p_1} + \frac{\partial \Pi_2}{\partial \omega}\frac{\partial \omega}{\partial p_1}\right) = \beta(1 - 2p_1 + (1-p_2)\,p_2)$$

and

$$\frac{d^2\Pi_1}{dp_1^2} = \beta\left(-2 - (2p_2 - 1)\frac{dp_2}{dp_1}\right) = \beta\left[-2 + \frac{\beta\left(2p_2-1\right)^2}{2\left(3p_2\left(1-\beta\right) + \beta p_1\right)}\right]$$

For a unique maximum it is sufficient to show that whenever the first order condition (FOC) is satisfied $(d\Pi_1/dp_1 = 0)$ then the second order condition (SOC) is satisfied $\left(d^2\Pi_1/dp_1^2 \leq 0\right)$. From the FOC, solve for p_1 and substituted into the SOC to yield:

$$\frac{d^2\Pi_1}{dp_1^2} = -2 + \frac{\beta\left(2p_2-1\right)^2}{2\left(3p_2\left(1-\beta\right) + \beta\frac{1}{2}\left(1 + \left(1-p_2\right)p_2\right)\right)} < 0$$

where the inequality follows given the feasible range for p_2. The bounds on p_1 can be obtained from the FOC and the bounds on p_2.
Revenue is increasing in β:

$$\frac{d\Pi_1\left(p_1^*, p_2^*\right)}{d\beta} = \frac{\partial \Pi_1}{\partial \beta} + \frac{\partial \Pi_1}{\partial p_1}\frac{\partial p_1}{\partial \beta} = p_1^*\left(1 - p_1^* + \left(1-p_2^*\right)p_2^*\right) - \left(1 - p_2^{*2}\right)p_2^* = \frac{1}{4}\left(-1 + p_2^* + \left(p_2^*\right)^2\right)^2 > 0$$

To obtain bounds on the optimal revenue,

$$\lim_{\beta \to 1} \Pi_1 = p_1\left(\frac{5}{4} - p_1\right) = 25/64$$

The other extreme yields

$$\lim_{\beta \to 0} \Pi_1 = \left(1 - p_2^{*2}\right)p_2^* = \frac{2}{3\sqrt{3}} = 0.3849$$

Finally, it is straightforward to show that the $\mathcal{U}_1 < \mathcal{U}_2$ constraint does not bind.
Social welfare is

$$S^* = \beta\left[\left(\frac{1-p_1^{*2}}{2}\right) + p_1^*\left(\frac{1-p_2^{*2}}{2}\right)\right] + (1-\beta)\left((1-p_2^*)\left(\frac{1+p_2^*}{2}\right) + p_2^*(1-p_2^*)\left(\frac{1+p_2^*}{2}\right)\right)$$

The first two terms are surplus when buyer A is informed and either purchases in period 1, or does not purchase in period 1 leaving the firm with the only option to sell to buyer B in period 2. The second two terms are surplus when buyer A is informed and the unit can be sold either to B or A in period 2.

□

Proof of Theorem 4. In period 2 the firm can only sell to buyer B. Hence, the optimal spot period price is $p_2^* = 1/2$ and the firm earns $\Pi_2^* = 1/4$. The uninformed buyer A purchases in advance only if $\mathcal{U}_1 \geq \mathcal{U}_2$. That constraint implies that period 1 price must satisfy: $p_1 \leq \frac{1+f^2}{2} - \frac{1}{16}$. The revenue function is strictly concave in p_1 and f but the unconstrained optimal values violates the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint. The firm's revenue function with the maximum p_1 is

$$\Pi_1 = \frac{1}{256}\left(16\left(7+4f-8f^2\right) - \beta\left(21+64f-176f^2+64f^4\right)\right)$$

It follows that

$$\frac{d\Pi}{df} = \frac{1}{8}\left(2(1-\beta) + 11\beta f - 8f\left(1+\beta f^2\right)\right)$$

Because $2(1-\beta) - 8\beta f^3$ is concave decreasing on the interval $[0,1]$, there is a unique $f \in [0,1]$ that maximizes Π_1.
Social welfare is

$$S = \beta\left((1-p_1)\left(\frac{1+p_1}{2}\right) + p_1(1-p_2)\left(\frac{1+p_2}{2}\right)\right) + (1-\beta)\left((1-f)\left(\frac{1+f}{2}\right) + f(1-p_2)\left(\frac{1+p_2}{2}\right)\right).$$

If Buyer A is informed, social welfare is equivalent to the base case with no recourse. If Buyer A is uninformed, Buyer A keeps the unit if $v_A \geq f$, and otherwise, the unit is transferred to Buyer B, if $v_B \geq p_2$.

□

Proof of Theorem 5. Advance Selling: In equilibrium p_1 is set so that the uniformed buyer A purchases in period 1. If there is a sale in period 1 then the firm is selling in period 2 to buyer B a unit it has already sold to A. If there is no sale in period 1, then buyer A must have been informed with a low value. That buyer does not remain in the market for period 2. So again, buyer B is the only customer in period 2. However, there are two different states in period 2. If a period 1 sale occurs, then the firm makes an offer to buyer B for a unit that it already sold, and therefore needs to provide buyer A with compensation b if the unit is transferred to buyer B. If a period 1 sale does not occur, then the firm is selling the unit it owns. In the former case the optimal period 2 price is $p_o = (1+b)/2$ and in the latter case the optimal price is $p_2 = 1/2$. If the uninformed A purchases in period 1 then he earns utility

$$\mathcal{U}_1 = \frac{1}{2}p_o + b(1-p_o) - p_1 = \frac{1}{4}\left(1+3b-2b^2\right) - p_1,$$

but if he waits (which is not the equilibrium), the firm assumes he was informed and so chooses $p_2 = 1/2$. The buyer's expected utility is $\mathcal{U}_2 = p_2(1-p_2)\left(\frac{1-p_2}{2}\right) = \frac{1}{16}$. The uniformed buyer A purchases in period 1 as long as

$$p_1 < \frac{1}{4}\left(1+3b-2b^2\right) - \frac{1}{16} = \frac{3}{16} + \frac{3b-2b^2}{4} \tag{4}$$

Given the period 2 prices, the firm's revenue is

$$\Pi_1 = \frac{1}{4}\left(1 + (1-\beta\tilde{v}_A)\left(4p_1 - 2b + b^2\right)\right)$$

The revenue function Π_1 increases in the period 1 price, p_1, and therefore the constraint binds. Given the constraint, the revenue function is

$$\Pi_1 = \frac{224 - 9\beta + 8(44-3\beta)b - 4\beta b^2 - 16(8-\beta)b^3}{512(1+b)}$$

and

$$\frac{d\Pi_1}{db} = \frac{128 - 15\beta - 8\beta b - 4(96-11\beta)b^2 - 32(8-\beta)b^3}{512(1+b)^2}$$

Some form of overbooking is optimal (i.e., $\partial\Pi_1/\partial b\,(b=1) < 0$). The optimal overbooking amount uniquely satisfies

$$128 - 15\beta = 8\beta b + 4(96-11\beta)b^2 + 32(8-\beta)b^3$$

Social welfare:

$$\begin{aligned} S &= \beta\left(\tfrac{1}{2}\tilde{v}_A\left(1-p_2^2\right) + (1-\tilde{v}_A)\left(\tfrac{1}{2}p_o(1+\tilde{v}_A) + \tfrac{1}{2}\left(1-p_o^2\right)\right)\right) + (1-\beta)\left(\tfrac{1}{2}p_o + \tfrac{1}{2}\left(1-p_o^2\right)\right) \\ &= \tfrac{1}{8}\left(5 + 2b\beta(1-\tilde{v}_A)\tilde{v}_A - 2\beta\tilde{v}_A^2 - (1-\beta\tilde{v}_A)b^2\right) \end{aligned}$$

If Buyer A is informed, he either: doesn't purchase and leaves the market if his valuation is low and the firm tries to sell to buyer B or he purchases in period 1 after which the firm tries to overbook and sell to Buyer B at price p_o. If Buyer A is uninformed, he purchases in period 1 and either keeps the unit (if the firm didn't succeed in selling to Buyer B), or loses the unit to Buyer B (if his value was higher than the overbooking price).
Price Skimming: There are four decisions for the firm. In period 1 the firm offers p_1 along with a buy back price of b. In period 2 the firm offers p_o if a sale occurred in period 1, otherwise p_2 is offered. The price p_o applies when a sale occurs in period 1 and only buyer B remains in period 2. To sell to buyer B requires the firm to pay b to buy back the unit from buyer A. Thus, to maximize its profit the firm offers buyer B the unit in period 2 at the price $p_o = (1+b)/2$. Now consider the other prices. In period 1 the informed buyer A's purchases when $v_A \geq \tilde{v}_A$, where

$$\tilde{v}_A = \frac{2p_1 - b + b^2}{1+b}.$$

Given the one-to-one relationship between $\tilde{v}_A$ and p_1, the firm's period 1 decision task to "select p_1 and b" can be reframed as "select $\tilde{v}_A$ and b" and the resulting period 1 price is

$$p_1 = \frac{1}{2}\left(b(1-b) + \tilde{v}_A(1+b)\right).$$

Say there isn't a sale in period 1. In equilibrium either buyer A was uninformed or he was informed but had a value too low to justify purchasing (i.e., $v_A < \tilde{v}_A$). In the first case, there are two buyers in period 2 whereas in the second case only buyer B remains in period 2. Let ω be the probability buyer A is in the market in period 2 (in equilibrium) conditional on there not being a sale in period 1:

$$\omega(\tilde{v}_A) = \frac{1-\beta}{1-\beta+\beta\tilde{v}_A}$$

The period 2 revenue function is

$$\Pi_2\left(p_2(\omega(\tilde{v}_A)), \omega(\tilde{v}_A)\right) = p_2(1-p_2)(1+\omega p_2)$$

Note that conditional on $\tilde{v}_A$, Π_2 does not depend on b. The optimal p_2 is

$$p_2(\omega) = \frac{-1+\omega+\sqrt{1+\omega+\omega^2}}{3\omega}$$

Note that $p_2(\omega)$ is increasing over the interval $\omega \in [0,1]$, with $\lim_{\omega\to 0} p_2(\omega) = 1/2$ and$p_2(1) = 1/\sqrt{3}$. Consider the period 1 decisions, $\tilde{v}_A$ and b. The firm's period 1 revenue is

$$\Pi_1(b, \tilde{v}_A) = \frac{1}{4}\beta(1+b)(1-\tilde{v}_A)(1-b+2\tilde{v}_A) + (1-\beta+\beta\tilde{v}_A)\Pi_2$$

The period 1 revenue is concave in b and maximized with $b = \tilde{v}_A$. The revenue function can therefore be written as

$$\Pi_1(\tilde{v}_A) = \frac{1}{4}\beta(1-\tilde{v}_A)(1+\tilde{v}_A)^2 + (1-\beta+\beta\tilde{v}_A)\Pi_2,$$

which is equivalent to the revenue function with price skimming in the reselling mechanism. Hence, overbooking and price skimming is equivalent (in equilibrium) to reselling and price skimming, i.e., same revenue and social welfare.

□

Proof of Theorem 6. No recourse: an uninformed buyer A's expected utility of waiting for period 2 is

$$\mathcal{U}_2 = p_2^2 \int_{p_2}^{1} (x - p_2)\, dx$$

where p_2^2 is the probability that the firm does not sell to the B buyers.
In an advance selling equilibrium, the firm needs to ensure that $\mathcal{U}_1 \geq \mathcal{U}_2$ so that an uninformed buyer A buys in period 1. Therefore, if there is no sale in period 1 the firm knows that the buyer A is informed and only buyers B remain on the spot. With a posted price mechanisms, period 2 revenue is

$$\Pi_2(p_2) = p_2\left(1 - p_2^2\right)$$

and period 1 revenue (assuming the constraint $\mathcal{U}_1 \geq \mathcal{U}_2$ is satisfied) is

$$\Pi_1 = p_1(1-\beta p_1) + \beta p_1 \Pi_2.$$

With advance selling, the second period revenue function is concave and maximized with price $p_2^* = \sqrt{1/3}$. The period 1 revenue function is concave in p_1. However, the unconstrained optimal price violates the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint. Given that constraint binds, the optimal period 1 price is $p_1^* = 1/2 - \left(2-\sqrt{3}\right)/9$. The firm's revenue is $\Pi_1^* = \left(90 + 36\sqrt{3} - 13\beta\right)/324$.
In a price skimming equilibrium, the uninformed buyer A does not purchase in period 1 because $\mathcal{U}_1 < \mathcal{U}_2$ is satisfied. Therefore, a sale occurs in period 1 only if buyer A is informed and has a high enough valuation. As in the case with a single buyer B, conditional on there no being a sale in period 1, the probability buyer A is in the market (i.e., he is uninformed) in period 2 is $\omega(p_1)$ given by equation (3). Period 2 revenue with 2 buyers B is

$$\Pi_2 = \omega(p_1)\, p_2\left(1 - p_2^3\right) + (1 - \omega(p_1))\, p_2\left(1 - p_2^2\right).$$

The first term is revenue when all three players remain in the market and the second term is revenue when only the two buyers B are in the market. Period 1 revenue is

$$\Pi_1 = \beta p_1(1-p_1) + (1 - \beta(1-p_1))\Pi_2^*.$$

With price skimming, in equilibrium the constraint $\mathcal{U}_1 < \mathcal{U}_2$ ensures that the uninformed buyer A does not purchase in period 1. The period 2 revenue function is concave in p_2, so the optimal p_2 solves the first-order-condition, $d\Pi_2/dp_2 = 0$. From the period 1 revenue function

$$\frac{d\Pi_1}{dp_1} = \frac{\partial \Pi_1}{\partial p_1} + (1-\beta+\beta p_1)\frac{\partial \Pi_2^*}{\partial \omega}\frac{\partial \omega}{\partial p_2} = \beta\left(1 - 2p_1 + p_2^*\left(1 - p_2^{*2}\right)\right)$$

and

$$\frac{d^2\Pi_1}{dp_1^2} = \beta\left(-2 + \left(1 - 3p_2^{*2}\right)\frac{dp_2^*}{dp_1}\right) = \beta\left(-2 + \left(3p_2^{*2} - 1\right)\frac{\partial p_2^*}{\partial \omega}\left(-\frac{\partial \omega}{\partial p_1}\right)\right)$$

To demonstrate that the FOC yields a unique and maximum solution, it is sufficient to demonstrate that for all solutions to the FOC, $\frac{d^2\Pi_1}{dp_1^2} \leq 0$. This follows because: $p_2^* \leq (1/2)^{2/3}$, $\partial p_2^*/\partial\omega \leq 1/4$, $-\partial\omega/\partial p_1 \leq 1$. It remains to confirm that the uninformed buyer A does not purchase in period 1. Not purchasing in period 1 yields the uninformed buyer A an expected utility of $p_2^{*2}\left(1 - p_2^*\right)^2/2$. Purchasing in period 1 yields expected utility $1/2 - p_1^*$. The uninformed buyer follows the equilibrium path if $p_1^* \geq \frac{1}{2}\left(1 - \left(1 - p_2^{*2}\right)p_2^{*2}\right)$, which holds given the equilibrium condition. Hence prices satisfy the following system of equations:

$$1 - 3\left(1 - \omega\right)p_2^{*2} - 4\omega p_2^{*3} = 0$$

$$p_1^* = \frac{1}{2}\left(1 + \left(1 - p_2^{*2}\right)p_2^*\right)$$

where ω is given by (3). The revenue function is increasing in β:

$$\frac{d\Pi_1^*}{d\beta} = \frac{\partial\Pi_1^*}{\partial\beta} = \left(1 - p_1^*\right)\left(p_1^* - \Pi_2^*\right) > 0$$

where the inequality follows from the derived bounds on prices.
As under both advance selling and price skimming we have that the period 2 revenue when there are 2 buyers B is greater than the period 2 revenue when there is a single buyer B for every p_2 and the period 1 revenue function structure is the same, so that the period 1 revenue with 2 buyers B is greater than the period 1 revenue with a single buyer, we must have that optimal revenues are higher with 2 buyers. Finally, to show that the price skimming equilibrium dominates, from the monotonicity in β, it is sufficient to show that the advance selling optimal revenue is smaller than the price skimming revenue at $\beta = 0$. For advance selling: $\Pi_1^*(0) = \left(90 + 36\sqrt{3}\right)/324 \approx 0.4702$. For price skimming: $\Pi_1^*(0) = \Pi_2^*(0) = \frac{3}{4}\left(\frac{1}{4}\right)^3 \approx 0.4725$.

Resale: As before, let $\mathcal{R}_A\left(p_r\right)$ be buyer A's utility conditional on owning the unit:

$$\mathcal{R}_A\left(p_r\right) = p_r^2 v_A + \left(1 - p_r^2\right)p_r.$$

To maximize expected utility, buyer A chooses the resale price $p_r^* = \left(v_A + \sqrt{3 + v_A^2}\right)/3$ and earns expected utility $\mathcal{R}_A^*\left(v_A\right) = \frac{1}{27}\left(v_A + \sqrt{3 + v_A^2}\right)\left(6 + v_A^2 + v_A\sqrt{3 + v_A^2}\right)$. In period 1 the informed buyer A earns utility $\mathcal{I}_1 = \mathcal{R}_A^* - p_1$ from purchasing the unit and therefore purchases the unit if $\mathcal{I}_1 \geq 0$. The expected utility $\mathcal{R}_A^*\left(v_A\right)$ is increasing in v_A implying that as in the single buyer B case, there exists a unique $\tilde{v}_A$ (possibly 0) above which an informed buyer A purchases in period 1. The uninformed buyer's expected utility from purchasing in period 1 is

$$\mathcal{U}_1 = \int_0^1 \mathcal{R}_A^*\left(x\right)dx - p_1 = \frac{1}{2} + \frac{\log 3}{8} - p_1.$$

Advance selling: With advance selling, as in the no-recourse case, if the firm does not sell in period 1, only the 2 buyers B remain in period 2. The optimal period 2 price is $p_2^* = \sqrt{1/3}$ and the period 2 revenue is $\Pi_2^* = \frac{2}{3\sqrt{3}}$. An uninformed buyer A's utility from deviating and waiting for period 2 is $\mathcal{U}_2 = p_2^2\int_{p_2}^1\left(x - p_2\right)dx = \frac{2-\sqrt{3}}{9}$. Therefore, for the uninformed A to purchase in period 1, the first period price must satisfy $p_1 \leq \frac{1}{2} + \frac{\log 3}{8} - \frac{2-\sqrt{3}}{9}$. The firm's period 1 revenue function given the constraint holds is

$$\Pi_1 = \left(1 - \beta\tilde{v}_A\right)p_1 + \beta\tilde{v}_A\Pi_2^*.$$

The firm's unconstrained revenue function is concave in p_1:

$$\frac{d\Pi_1}{dp_1} = \frac{\partial\Pi_1}{\partial p_1} + \frac{\partial\Pi_1}{\partial\tilde{v}_A}\frac{\partial\tilde{v}_A}{\partial p_1} = 1 - \beta\left(\tilde{v}_A + p_1 - \Pi_2^*\right)$$

and

$$\frac{d\Pi_1^2}{dp_1^2} = -\beta\left(1+\frac{\partial\tilde{v}_A}{\partial p_1}\right) < 0$$

where the inequality follows because $\tilde{v}_A$ is an increasing function of p_1. However, in equilibrium we must have that $\mathcal{U}_1 \geq \mathcal{U}_2$ and hence $p_1 \leq \frac{1}{2}+\frac{\log 3}{8}-\frac{2-\sqrt{3}}{9}$. Substituting $p_1 = \frac{1}{2}+\frac{\log 3}{8}-\frac{2-\sqrt{3}}{9}$ and the corresponding $\tilde{v}_A = 0.496$, we obtain $d\Pi_1/dp_1 > 0\ \forall\beta$ and therefore the constraint binds. The revenue function increases in β :

$$\frac{d\Pi_1}{d\beta} = -\tilde{v}_A\left(p_1^* - \Pi_2^*\right) < 0.$$

Price skimming: With price skimming, only the informed buyer A with value greater than $\tilde{v}_A$ considers purchasing in period 1. This threshold influences the probability ω as in equation (1). Conditional on owning a unit, the firm's revenue function in period 2 is

$$\Pi_2 = \omega\left(1-p_2^3\right)p_2 + (1-\omega)\left(1-p_2^2\right)p_2.$$

The period 1 revenue function for the firm is

$$\Pi_1 = \beta\left(1-\tilde{v}_A\right)p_1 + \left(1-\beta\left(1-\tilde{v}_A\right)\right)\Pi_2^*.$$

The period 2 revenue function is concave in p_2, so the optimal p_2 solves the first-order-condition, $d\Pi_2/dp_2 = 0$. Given $\omega \in [0,1]$, it follows that $p_2^* \in \left[1/\sqrt{3}, (1/2)^{2/3}\right]$. Because there is a one-to-one correspondence between $\tilde{v}_A$ and p_1, the equilibrium can be defined in terms of $\tilde{v}_A$ instead of p_1. The revenue function is:

$$\Pi_1 = \beta\left(1-\tilde{v}_A\right)\mathcal{R}_A\left(\tilde{v}_A\right) + \left(1-\beta+\beta\tilde{v}_A\right)\Pi_2^*.$$

Differentiating the period 1 revenue function

$$\begin{aligned}\frac{d\Pi_1}{d\tilde{v}_A} &= \frac{\partial\Pi_1}{\partial\tilde{v}_A} + \left(1-\beta\left(1-\tilde{v}_A\right)\right)\left(\frac{\partial\Pi_2^*}{\partial\omega}\frac{\partial\omega}{\partial\tilde{v}_A}\right). \\ &= \beta\left[-\mathcal{R}_A\left(\tilde{v}_A\right) + \left(1-\tilde{v}_A\right)\frac{d\mathcal{R}_A}{d\tilde{v}_A} + \left(1-p_2^{*2}\right)p_2^{*2}\right]\end{aligned}$$

Solving the FOC yields an implicit solution for $\tilde{v}_A^*$. To demonstrate that the FOC yields a unique and maximum solution, it is sufficient to show that for all solutions to the FOC $d\Pi_1^2/d^2\tilde{v}_A \leq 0$. This follows because: $p_2^* \leq (1/2)^{2/3}$, $\partial p_2^*/\partial\omega \leq 1/4$, $-\partial\omega/\partial\tilde{v}_A \leq 1$, and $-\mathcal{R}_A\left(\tilde{v}_A\right) + \left(1-\tilde{v}_A\right)\frac{d\mathcal{R}_A}{d\tilde{v}_A} \leq -\frac{2}{3}\left(1-\frac{1}{\sqrt{3}}\right)$. Therefore, with price skimming $p_1^* = \mathcal{R}_A^*\left(\tilde{v}_A^*\right)$,

$$\begin{aligned}1-3\left(1-\omega\right)p_2^{*2} - 4\omega p_2^{*3} &= 0 \\ p_1^* &= \frac{1}{2}\left(1+\left(1-p_2^*\right)p_2^*\right)\end{aligned}$$

where ω is given by (3). It remains to confirm that the uninformed buyer A does not purchase in period 1. Waiting for period 2 yields the uninformed buyer A an expected utility of $p_2^{*2}\left(1-p_2^*\right)^2/2$. Purchasing in period 1 yields expected utility $\frac{1}{2}+\frac{\log 3}{8}-p_1$. The uninformed buyer A follows the equilibrium path if

$$p_1^* \geq \frac{1}{2}+\frac{\log 3}{8}-\frac{p_2^{*2}\left(1-p_2^*\right)^2}{2},$$

which follows from the equilibrium conditions. The revenue function is increasing in β :

$$\frac{d\Pi_1^*}{d\beta} = \left(1-\tilde{v}_A^*\right)\left(p_1^*-\Pi_2^*\right) > 0,$$

where the inequality follows from the derived bounds on prices.

□

B Second Period Auction

Assume that if both buyers are present in period 2 and the firm is still offering the unit for sale, then the firm runs an optimal auction to try and sell the unit. Consequently, an uninformed buyer A is first given an opportunity to purchase the item in period 1 (before v_A is known) and then has a second opportunity in period 2 (after v_A is observed), but risks not being able to purchase the unit in period 2 if buyer B wins the auction.

Prior to examining the different mechanisms, it is instructive to establish the firm's optimal strategy in the second period and the uninformed buyer A's expected utility of not buying in period 1. The optimal mechanism is a 2nd price auction with reservation price. In particular, for n buyers with symmetric valuations $U[0,1]$, the firm's optimal reservation price is $z = 1/2$ independent on the number of buyers n.

Lemma 1. *Suppose there are n buyers with valuations distributed $U[0,1]$ and a firm with one unit selling in one period. The mechanism that maximizes the firm's expected revenue is a second price auction with reservation price $z = 1/2$. The firm's expected revenue is*

$$\pi(n) = n\left((1-z)z^n + (n-1)\int_z^1 (1-y)y^{n-1}dy\right) = \frac{n+2^{-n}-1}{n+1} \tag{5}$$

and a buyer's expected utility is

$$\mathcal{U}(n) = \int_z^1 \left((x-z)z^{n-1} + (n-1)\int_z^x (x-y)y^{n-2}dy\right)dx = \frac{2(2^n-1)-n}{2^{n+1}n(n+1)}. \tag{6}$$

Proof. For the proof that a second price auction with a reservation price maximizes the seller's revenue see Myerson (1981). For the derivation of the firm's revenue and a buyer's expected utility, note that from the revelation principle that bidders bids their true value. Let $G(\cdot)$ be the distribution of the maximum value of $n-1$ bidders and $g(\cdot)$ be the probability distribution function. If the sellers sets a positive reservation price $z > 0$, a bidder may only win if $v \geq z$. For $v \geq z$, a bidder expected payment to the seller is

$$\pi(v,z) = zG(z) + \int_z^v yg(y)\,dy.$$

The firm does not know bidder's value, but knows that values are distributed $F(\cdot)$. Hence, the firm's expected profit is

$$\begin{aligned}\pi &= n\int_z^1 \pi(v,z)f(v)\,dv = nz\bar{F}(z)G(z) + n\int_z^1\left(\int_z^x yg(y)\,dy\right)f(y)\,dx \\ &= nz\bar{F}(z)G(z) + n\int_z^1 \bar{F}(y)yg(y)\,dy.\end{aligned}$$

A buyer's expected utility is

$$\mathcal{U}(z) = \int_z^1\left((x-z)G(z) + \int_z^x (x-y)g(y)\,dy\right)dx.$$

For the uniform distribution on $[0,1]$, $F(x) = x$ and $G(x) = x^{n-1}$. Plugging in these distribution and the optimal $z = 1/2$, we get the desired result.

□

Using the derivations in the lemma, we obtain the following result:

Theorem 7. *If the firm runs an optimal auction in period 2, it sets a reservation price $p_2^* = 1/2$ in the second period. Furthermore, the following hold:*

- *No recourse: there exists a unique advance selling equilibrium with first period price $p_1^* = 5/12$ and revenue $\Pi_1^* = \frac{5}{72}(6-\beta)$ and a unique price skimming equilibrium with $p_1^* = 5/8$ and revenue $\Pi_1^* = \frac{5}{192}(16-\beta)$. Price skimming dominates advance selling for all β.*
- *Resale: there exists a unique advance selling equilibrium with price $p_1^* = 1/2$ and revenue, $\Pi_1^* = \left(2-\left(\sqrt{2}-1\right)\beta\right)/4$ and a unique price skimming equilibrium with price $p_1^* = \left(11+4\sqrt{7}\right)/36$ and revenue $\Pi_1 = \frac{5}{12} - \frac{19-7\sqrt{7}}{54}\beta$.*
- *Refund/Options: there exists a unique advance selling equilibrium with price $p_1^* = \frac{1}{2}\left(1+f^2\right) - 1/12$, where f is the unique solution to $3(1-\beta) - 12\beta f^3 - (12-17\beta) f = 0$. The price skimming equilibrium is equivalent to the price skimming equilibrium with no recourse.*
- *Overbooking: there exists a unique advance selling equilibrium with $p_1^* = \frac{1}{6} + \frac{3b^* - 2b^{*2}}{4}$, $p_h^* = (1+b^*)/2$ and the overbooking payment, $b^* < 1$, is the solution to*

$$4(9-\beta) = 3\beta b^* + 12(9-\beta) b^{*2} + 9(8-\beta) b^{*3}$$

and a unique price skimming equilibrium with $p_h^* = (1+b^*)/2$, $b^* = p_1^* = \tilde{v}_A^*$, and $\tilde{v}_A^* = \frac{1}{3}\left(\sqrt{7}-1\right)$. Revenue in this equilibrium are equivalent to the revenue and surplus in the price skimming equilibrium with the reselling mechanism.

Proof. No recourse: The expected utility of not buying in period 1 is $\mathcal{U}_2 = \mathcal{U}(2) = \frac{1}{12}$, from (6). In the advance selling equilibrium the uninformed buyer A purchases in period 1 because in equilibrium $\mathcal{U}_2 \leq \mathcal{U}_1$. Thus, if there is no sale in period 1, it must be that buyer A was informed and has a low valuation, $v_A < p_1$. In that case, only buyer B remains in period 2. The firm sets a period 2 price $p_2^* = z^* = 1/2$, the period 2 revenue is $\Pi_2^*(p_2) = 1/4$ and period 1 revenue (assuming the constraint $\mathcal{U}_2 \leq \mathcal{U}_1$ is satisfied) is $\Pi_1(p_1) = (1-\beta p_1) p_1 + \beta p_1 \Pi_2^*$. The period 1 revenue function is concave in p_1. However, the unconstrained optimal price violates the $\mathcal{U}_2 \leq \mathcal{U}_1$ constraint. Given that constraint binds, the optimal period 1 price is $p_1^* = 7/16$. With price skimming, period 2 revenue is

$$\Pi_2^* = \omega(p_1)\pi(2) + (1-\omega(p_1))\pi(1) = \frac{1}{4} + \frac{1}{6}\omega$$

where $\pi(n)$ is the expected revenue from running the optimal auction in period 2 with n buyers (given in equation (5)). The first term is revenue when both buyers remain in the market and the second term is revenue when only buyer B is present. Period 1 revenue is $\Pi_1(p_1) = \beta(1-p_1)p_1 + (1-\beta+\beta p_1)\Pi_2^*$. The period 1 revenue function in concave in p_1 and maximized at $p_1^* = 5/8$. To ensure that uninformed buyer A does not purchase in period 1, we must have $\mathcal{U}_1 \leq \mathcal{U}_2$. The constraint is not binding in equilibrium and therefore the optimal period 1 price is $p_1^* = 5/8$ and the resulting profit is $\Pi_1^* = \frac{5}{192}(16-\beta)$.
Resale: With advance selling while in equilibrium the uninformed buyer A is expected to purchase in period 1, the buyer does have the option to wait to try to purchase in period 2. Doing so yields expected utility $\mathcal{U}_2$, where $\mathcal{U}_2 = \mathcal{U}(2) = \frac{1}{12}$, which is the expected utility if the firm runs an optimal auction with 2 buyers. The uninformed buyer A purchases in the advance period if $\mathcal{U}_2 \leq \mathcal{U}_1$, which requires$p_1 \leq \frac{7}{12} - \frac{1}{12} = \frac{1}{2}$. The firm's period 1 revenue function (assuming the $\mathcal{U}_2 \leq \mathcal{U}_1$ constraint is satisfied) is $\Pi_1 = (1-\beta\tilde{v}_A) p_1 + \beta\tilde{v}_A \Pi_2^*$. The firm's revenue function is increasing in p_1. Thus, the firm chooses the maximum period 1 price that satisfies

the $\mathcal{U}_2 \leq \mathcal{U}_1$ constraint. With price skimming, The firm's revenue function in period 2 (conditional on still owning the unit) is

$$\Pi_2^* = \omega(\tilde{v}_A)\pi(2) + (1-\omega(\tilde{v}_A))\pi(1) = \frac{1}{4} + \frac{1}{6}\omega \tag{7}$$

with probability ω the firm runs an auction to sell to two consumers (B and A), and with probability $1-\omega$ the firm is posting a price to sell only to one buyer (B). The period 1 revenue function for the firm is

$$\Pi_1 \;=\; \beta(1-\tilde{v}_A)p_1 + (1-\beta+\beta\tilde{v}_A)\,\Pi_2$$

Because there is a one-to-one relationship between $\tilde{v}_A$ and p_1, the equilibrium can be defined in terms of $\tilde{v}_A$ instead of p_1. The revenue function is

$$\Pi_1 \;=\; \tfrac{1}{4}\beta(1-\tilde{v}_A)(1+\tilde{v}_A)^2 + (1-\beta+\beta\tilde{v}_A)\,\Pi_2^*$$

which is concave in $\tilde{v}_A$. Solving the first-order condition (FOC), $d\Pi_1/d\tilde{v}_A = 0$, yields

$$\tilde{v}_A^* = \frac{\sqrt{7}-1}{3}$$

and

$$p_1^* = \frac{(1+\tilde{v}_A^*)^2}{4} = \frac{11+4\sqrt{7}}{36}.$$

It remains to confirm that the uniformed buyer A indeed does not purchase in period 1. Not purchasing in period 1 yields the uninformed buyer A an expected utility of $\mathcal{U}(2) = 1/12$. Purchasing in period 1 yields expected utility $7/12 - p_1^*$. The uninformed buyer A follows the equilibrium path if $p_1^* \geq 1/2$, which holds. Refund: An uniformed buyer A can choose not to purchase in period 1 and earn utility $\mathcal{U}_2 = \mathcal{U}(2) = 1/12$. With advance selling, the firm starts period 2 with a unit only if it was purchased by an uninformed buyer A in period 1 and returned or it was not purchased by an informed buyer A in period 1. Either way, the only possible customer in period 2 is buyer B and the optimal period 2 price is $p_2^* = 1/2$. The firm's period 1 revenue is

$$\Pi_1 = \beta\left((1-p_1)\,p_1 + p_1/4\right) + (1-\beta)\left(p_1 - f^2 + f/4\right).$$

In period 2 the firm can only sell to buyer B. Hence, the optimal spot period price is $p_2^* = 1/2$ and the firm earns $\Pi_2^* = 1/4$. The uninformed buyer A purchases in advance if $\mathcal{U}_1 \geq \mathcal{U}_2$. That constraint implies $p_1 \leq \frac{1+f^2}{2} - \frac{1}{12}$. The revenue function is strictly concave in p_1 and f but the unconstrained optimal values violate the $\mathcal{U}_1 \geq \mathcal{U}_2$ constraint. The firm's revenue function with the maximum p_1 is

$$\Pi_1 = \frac{1}{72}\left(6\left(5+3f-6f^2\right) - \beta\left(5+18f-51f^2+18f^4\right)\right)$$

It follows that

$$\begin{aligned}\frac{d\Pi}{df} &= \frac{1}{12}\left(3-12f-\beta\left(3-17f+12f^3\right)\right)\\ &= \frac{1}{12}\left(3\left(1-\beta\right) - 12\beta f^3 - f\left(12-17\beta\right)\right)\end{aligned}$$

Because $3\left(1-\beta\right) - 12\beta f^3$ is concave decreasing on the interval $[0,1]$,there is a unique $f \in [0,1]$ that maximizes Π_1.

Overbooking: With advance selling, p_1 is chosen so that the uniformed buyer A purchases in period 1. The firm's revenue is

$$\Pi_1 = \beta\tilde{v}_A\pi(1) + (1-\beta\tilde{v}_A)\left(p_1p_h + (p_1 - b + p_h)(1-p_h)\right)$$

In equilibrium p_1 is set so that the uniformed buyer A purchases in period 1. If there is a sale in period 1 then the firm is selling in period 2 to buyer B a unit it has already sold to A. If there is no sale in period

1, then buyer A must have been informed with a low value. That buyer does not remain in the market for period 2. So again, buyer B is the only customer in period 2. However, there are two different states in period 2. If a period 1 sale occurs, then the firm makes an offer to buyer B for a unit that it already sold, and therefore needs to provide buyer A with compensation b if the unit is transferred to buyer B. If a period 1 sale does not occur, then the firm is selling the unit it owns through an auction. In the former case the optimal period 2 price is $p_h = (1+b)/2$ and in the latter case the reservation price is $z = 1/2$. If the uninformed A purchases in period 1 then he earns utility

$$\mathcal{U}_1 = \frac{1}{2}p_h + b(1-p_h) - p_1 = \frac{1}{4}\left(1+3b-2b^2\right) - p_1$$

but if he waits (which is not the equilibrium), the buyer's expected utility is $\mathcal{U}_2 = \frac{1}{12}$. The uniformed buyer A purchases in period 1 as long as

$$p_1 \leq \frac{1}{4}\left(1+3b-2b^2\right) - \frac{1}{12} = \frac{1}{6} + \frac{3b-2b^2}{4}$$

Given the period 2 auction, the firm's revenue is

$$\Pi_1 = \frac{1}{4}\left(1 + (1-\beta\tilde{v}_A)\left(4p_1 - 2b + b^2\right)\right)$$

It can be shown that the p_1 constraint binds. Given the constraint, the revenue function is

$$\Pi_1 = \frac{4(30-\beta) + 12(16-\beta)b - 3\beta b^2 - 9(8-\beta)b^3}{288(1+b)}$$

and

$$\frac{d\Pi_1}{db} = \frac{4(9-\beta) - 3\beta b - 12(9-\beta)b^2 - 9(8-\beta)b^3}{144(1+b)^2}$$

Some form of overbooking is optimal (i.e., $\partial\Pi_1/\partial b\,(b=1) < 0$). The optimal overbooking amount uniquely satisfies

$$4(9-\beta) = 3\beta b + 12(9-\beta)b^2 + 9(8-\beta)b^3$$

where uniqueness is guaranteed because the RHS of the equation is increasing in b, while the LHS is constant.

□

C Proofs of Extensions and Additional Results

In this appendix we elaborate on several extensions to confirm the robustness of the results. In particular, we analyzed the following: (i) an informed buyer A is forward-looking; (ii) random allocation of the unit in period 2. Both extensions illustrate that the results are robust to other model specifications, even though the analysis is more tedious.

C.1 Informed Buyer A is Patient

Our model assumes that an informed Buyer A that does not purchase in period 1 exits the market. We analyze a version of the model in which the informed buyer is patient and considers waiting to period 2. While this model is more complex, because it gives rise to more cases to consider, the analysis illustrates that the results are qualitatively similar. Specifically, forward-looking behavior does not affect the advance selling equilibrium revenue–an informed buyer that expects an increase in price does not have an incentive to wait, but increases the price-skimming revenue because of the negative effect of strategic waiting. Therefore, the range of βs for which advance selling dominates price skimming increases. Furthermore, we find that as in the original model, overbooking and resale are equivalent with when combined with price skimming.

To facilitate the analysis, observe that there are four cases to consider in each model: (1) increasing price path with the uninformed buyer A buying in advance; (2) decreasing price path with the uninformed buyer A buying in advance; (3) increasing price path with the uninformed buyer A waiting; and (4) decreasing price path with the uninformed buyer A waiting.

C.1.1 No recourse strategy

Begin with finding equilibria in which the uninformed buyer A purchases in period 1 (cases (1) and (2)) For an uninformed buyer A to buy in advance it must be that $\mathcal{U}_1 \geq \mathcal{U}_2$, which implies that prices are such that $\frac{1}{2} - p_1 \geq \frac{1}{2} p_2 (1-p_2)^2$. In case (1), the price path is increasing, so an informed buyer A has no incentive to wait and the equilibrium prices and revenue are the same as in Theorem 3.

The fact that the informed buyer A is forward-looking gives rise to case (2), in which prices are decreasing and an informed buyer A may choose to wait. In particular, in this case, if there is no sale in period 1, it must be that buyer A was informed and chose to wait. Therefore, in that case, the firm knows that in period 2 both buyer B and an informed buyer A with low value remain. An informed buyer A with value v_A gets utility $v_A - p_1$ from buying the period 1 and utility $p_2 (v_A - p_2)$ from waiting. For every set of prices (p_1, p_2) there exists a unique $\hat{v}_A = \min\left\{\frac{p_1 - p_2^2}{1-p_2}, 1\right\}$ so that if $v_A \geq \hat{v}_A$ the customer buys in period 1 and if $v_A < \hat{v}_A$, she waits and that $\hat{v}_A > p_1$. Period 2 revenue is therefore

$$\Pi_2 = \left(1 - p_2 + p_2\left(\frac{\hat{v}_A - p_2}{\hat{v}_A}\right)\right) p_2 = p_2\left(1 - \frac{p_2^2}{\hat{v}_A}\right),$$

which is complicated by the fact that an informed buyer A that didn't purchase in period 1 is in the market in period 2. Period 1 revenue is

$$\Pi_1 = (1 - \beta + \beta (1 - \hat{v}_A)) p_1 + \beta \hat{v}_A \Pi_2.$$

The first term is when the informed buyer A purchases in period 1 or buyer A is uninformed and the second term is the expected revenue from period 2 when there is no sale in period 1.

Next, we examine cases (3) and (4) in which an uninformed buyer A waits, i.e., $\mathcal{U}_1 < \mathcal{U}_2$. In these cases, the firm knows that a sale may occur in period 1 only the buyer A was informed and her value was sufficiently high, $v_A \geq \hat{v}_A$. With an increasing price path (case (3)), an informed buyer doesn't wait. Period 2 revenue is

$$\Pi_2 = \omega(p_1)\left(1 - p_2^2\right) p_2 + (1 - \omega(p_1)) p_1 (1 - p_2) p_2,$$

where $\omega(p_1)$ defined as in the base model. Period 1 revenue is

$$\Pi_1 = \beta (1 - p_1) p_1 + (1 - \beta (1 - p_1)) \Pi_2^*.$$

With a decreasing price path (case (4)), an informed buyer waits if $v_A < \hat{v}_A$. Let $\omega(\hat{v}_A)$ be the probability that buyer A was uninformed in period 1, conditional on there not being a sale in period 1:

$$\omega(\hat{v}_A) = \frac{1-\beta}{1 - \beta(1 - \hat{v}_A)}.$$

Then, period 2 revenue is

$$\begin{aligned}\Pi_2 &= \omega(\hat{v}_A)(1 - p_2 + p_2(1-p_2)) p_2 + (1 - \omega(\hat{v}_A))\left(1 - p_2 + p_2\left(1 - \frac{p_2}{\hat{v}_A}\right)\right) p_2 \\ &= \left(1 - \frac{p_2^2}{1 - \beta(1 - \hat{v}_A)}\right) p_2.\end{aligned}$$

The following establishes the no recourse equilibrium.

Theorem 8. *In the no-recourse mechanism there exists a threshold β^o, such that: (i) if $\beta \leq \beta^0$, the firm implements an advance selling strategy with prices $p_1^* = 7/16$, $p_2^* = 1/2$. The firm's revenue is $\Pi_1^* = (7/256)(16 - 3\beta)$. This equilibrium is equivalent to the no-recourse equilibrium in which the informed buyer A is impatient: prices are increasing, an uninformed buyer A purchases in period 1 and an informed buyer A does not wait for period 2; (ii) otherwise, the firm implements a price skimming equilibrium. Prices satisfy the following system of equations:*

$$p_1^* = \hat{v}_A - p_2^* \left(\hat{v}_A - p_2^*\right)$$

$$p_2^* = \frac{\sqrt{1 - \beta\left(1 - \hat{v}_A^*\right)}}{\sqrt{3}},$$

where $\hat{v}_A^*$ is the solution to

$$5\sqrt{3}\beta\hat{v}_A^2 + 4(1+\beta)\sqrt{1-\beta(1-\hat{v}_A)} = \left(4\sqrt{3} - 5\sqrt{3}\beta + (12+4\beta)\sqrt{1-\beta(1-\hat{v}_A)}\right)\hat{v}_A.$$

In this equilibrium, prices are decreasing, an uninformed buyer A waits for period 2 and an informed buyer A may wait for period 2. Furthermore, the firm's revenue in this case is higher than the revenue with price skimming in which the informed buyer A is impatient.

Proof. Advance selling: First, analyze case (2). Taking the second derivative of Π_2, we get $\Pi_2''(p_2) = -6p_2/\hat{v}_A < 0$. Therefore, Π_2 is concave and the optimal period 2 price is $p_2^* = \sqrt{\hat{v}_A/3}$ and $\Pi_2^* = 2\sqrt{\hat{v}_A/3}/3$. From the monotonicity between $p_1$and $\hat{v}_A$, we can write Π_1 in terms of $\hat{v}_A$ alone:

$$\Pi_1(\hat{v}_A) = \frac{\hat{v}_A}{9}\left(12 - \sqrt{3}(3-2\beta)\sqrt{\hat{v}_A} - 12\beta\hat{v}_A + 3\sqrt{3}\beta\hat{v}_A^{3/2}\right).$$

Π_1 is concave in $\hat{v}_A$. For $\mathcal{U}_1 \geq \mathcal{U}_2$, we must have that $\hat{v}_A \leq \bar{v}_A < 1$, where $\bar{v}_A$ is the solution to $25\bar{v}_A^3 - 138\bar{v}_A^2 + 117\bar{v}_A - 27 = 0$. Evaluating $\Pi_1'(\hat{v}_A = \bar{v}_A)$, we get that $\Pi_1'(\hat{v}_A = \bar{v}_A) > 0$, which implies that the solution to the constrained problem is $\hat{v}_A^* = \bar{v}_A \approx 0.4522$. Plugging it into the revenue function and comparing with the revenue function of case (1), we get that case (1) dominates case (2) for every β.

The theorem establishes that case (2) is dominated by case (1) and therefore a firm who's strategy is to sell to an uninformed buyer in period 1 adopts an increasing price path and gives up the opportunity to sell to an informed buyer A in period 2.

Price Skimming: First, analyze case (3). Differentiating Π_2 with respect to p_2 twice, we get

$$\frac{d^2\Pi_2}{dp_2^2} = -\frac{2\beta p_1 + 6p_2(1-\beta)}{1-\beta(1-p_1)} < 0$$

and therefore, $\Pi_2(p_2)$ is concave. Solving $\Pi_2'(p_2) = 0$, we get that the optimal period 2 price is

$$p_2^* = \frac{-2\beta p_1 + \sqrt{4\beta^2 p_1^2 + 12(1-\beta)(1-\beta(1-p_1))}}{6(1-\beta)}.$$

Plugging p_2^* into Π_1 we get a function in p_1 alone, which is strictly concave if $\beta \leq \frac{12}{143}\left(24 - 7\sqrt{3}\right) \approx 0.9966$ and unimodal (convex-concave) otherwise. It therefore has a unique maximum. For the price path to be increasing, the constraint $p_1 \leq p_2$ implies that we must have

$$p_1 \leq \bar{p}_1 = \frac{\beta + \sqrt{12 - 16\beta + 5\beta^2}}{2(3-\beta)}.$$

Plugging $p_1 = \bar{p}_1$ into $d\Pi_1/dp_1$, we get that $d\Pi_1/dp_1\mid_{p_1=\bar{p}_1} > 0$ and therefore the optimal advance and spot period prices must be $p_1 = p_2 = \bar{p}_1$.

Next, analyze case (4). Solving $d\Pi_2^{s,w}/dp_2 = 0$, we find that the optimal spot period price is:

$$p_2(\hat{v}_A) = \min\left\{\hat{v}_A, \sqrt{\frac{1-\beta(1-\hat{v}_A)}{3}}\right\}$$

and that the solution is interior where $\hat{v}_A \geq \frac{1}{6}\left(\beta + \sqrt{12(1-\beta)+\beta^2}\right)$. Assume that the solution is interior. Then, writing the optimal spot period revenue as a function of $\hat{v}_A$, we get: $\Pi_2^{s,w} = 2\sqrt{1-\beta(1-\hat{v}_A)}/3\sqrt{3}$ and the total revenue is: $\Pi_1^{s,w}(p_2; p_1, \hat{v}_A) = \beta(1-\hat{v}_A)p_1 + (1-\beta(1-\hat{v}_A))\Pi_2^{s,w}$. For a given $\hat{v}_A$, the firm wishes to maximize p_1, because the optimal p_2 does not depend on p_1 given $\hat{v}_A$, so we have $p_1(\hat{v}_A) = \hat{v}_A - p_2(\hat{v}_A)(\hat{v}_A - p_2(\hat{v}_A))$. Plugging $p_1(\hat{v}_A)$ into the revenue function we get:

$$\Pi_1^{s,w}(\hat{v}_A) = \beta(1-\hat{v}_A)(\hat{v}_A - p_2(\hat{v}_A - p_2)) + (1-\beta(1-\hat{v}_A))\Pi_2^{s,w}(\hat{v}_A),$$

where $p_2 = \sqrt{(1-\beta(1-\hat{v}_A))/3}$. Differentiating $\Pi_1^{s,w}(\hat{v}_A)$ as a function of $\hat{v}_A$ twice, we get that the function is strictly concave, so uniqueness of $\hat{v}_A$ is guaranteed. To verify that $\hat{v}_A$ is indeed interior, note that

$$\frac{d\Pi_1^{s,w}(\hat{v}_A)}{d\hat{v}_A}\Big|_{\hat{v}_A=\frac{1}{6}\left(\beta+\sqrt{12(1-\beta)+\beta^2}\right)} > 0 \quad \forall\beta.$$

Let $\hat{v}_A^e$, p_1^e and p_2^e be the equilibrium threshold and prices. Finally, we confirm that the equilibrium prices satisfy both price skimming and that the uninformed A waits for the spot period, i.e., $p_1^e > \max\left\{p_2^e, \frac{1}{2}\left(1 - p_2^e(1-p_2^e)^2\right)\right\}$. To check price-skimming in equilibrium, note that $p_1^e = \hat{v}_A^e - p_2^e(\hat{v}_A - p_2^e)$ and $p_2^e = \sqrt{(1-\beta(1-\hat{v}_A^e))/3}$. Plugging in and simplifying, we get that $p_1^e > p_2^e$ for all $\hat{v}_A \geq \frac{1}{6}\left(\beta + \sqrt{12(1-\beta)+\beta^2}\right)$, so this is indeed a price-skimming equilibrium. To verify that $p_1^e > \frac{1}{2}\left(1 - p_2^e(1-p_2^e)^2\right)$, observe that $\frac{d\Pi_1^{s,w}(\hat{v}_A)}{d\hat{v}_A}\Big|_{\hat{v}_A=\sqrt{1/3}} > 0 \;\; \forall\beta$ and therefore $\hat{v}_A^e > \sqrt{1/3}$. Since $p_1(\hat{v}_A) > \frac{1}{2}\left(1 - p_2(\hat{v}_A)(1-p_2(\hat{v}_A))^2\right)$ for all $\hat{v}_A > \sqrt{1/3}$, the uninformed A waits in equilibrium. Finally, comparing the revenue functions of the two cases, we find that case (4) always dominates case (3) and hence the equilibrium in which an uninformed buyer A waits involves a decreasing price path.

□

In the region in which advance selling is the optimal price path strategy revenue is equivalent whether the informed buyer A is patient or impatient. However, optimal price skimming revenue is higher than the revenue in which the informed buyer A leaves the market in period 2, as Theorem 8 illustrates. To explain, if there were no sale in period 1, the firm faces two buyers in period 2, which is better than having a single buyer. It is therefore able to charge higher prices and improve revenue. Hence, having a forward-looking informed buyer A improves the price skimming strategy over the advance selling one (i.e., price skimming dominates over a larger range of βs).

C.1.2 Resale

We reach a similar conclusion with a resale mechanism. Again, an informed buyer does not wait if the price path is increasing, so focusing on an increasing price path, the analysis is the same as in the original model. In fact, the following result demonstrates that as in the original model, with resale the only equilibrium sustainable involves a declining price path.

Theorem 9. *With resale, there exists a unique β^r, so that (i) if $\beta \leq \beta^r$, the firm implements an advance-selling strategy, equivalent to the equilibrium in which the informed buyer A is impatient. The firm sets prices $p_1^* = 25/48$ and $p_2^* = 1/2$, an uninformed buyer A purchases in advance and an informed buyer A does not wait. The firm's optimal revenue is $\Pi_1^* = \left(150 - \left(65\sqrt{3} - 78\right)\beta\right)/288$; (ii) otherwise, the firm implements a price-skimming strategy in which prices satisfy the following system of equations:*

$$p_1^* = \frac{(1+\hat{v}_A^*)^2}{4} - p_2^*(\hat{v}_A^* - p_2^*), \quad p_2^* = \sqrt{\frac{1-\beta(1-\hat{v}_A^*)}{3}},$$

an uninformed buyer A waits for period 2 and an informed buyer A waits if $v_A \geq \hat{v}_A^*$, where $\hat{v}_A^*$ is the solution larger solution to

$$(1-8\beta)\,z + 2\hat{v}_A\left(-4\sqrt{3}+5\sqrt{3}\beta+(3+4\beta)\,z\right)+\hat{v}_A^2\left(-10\sqrt{3}\beta+9z\right)=0$$

and $z = \sqrt{1-\beta\,(1-\hat{v}_A)}$. The revenue in this case is higher than the price skimming revenue with an impatient informed buyer A.

Proof. Consider first the case in which the firm adopts an increasing price path and hence a forward-looking buyer A does not wait. This case is no different than the behavior of the original model and we showed that an increasing price path does not happen in equilibrium. It is easy to show that it does not happen in equilibrium here either: restricting prices to this case results in the firm dynamically setting advance and spot period prices $p_1 = p_2 = 1/2$ and the uninformed buyer A purchases in advance, which is dominated by the decreasing price path advance selling equilibrium.

Next, consider the case in which the firm adopts a decreasing price path. Under price-skimming with resale, an informed buyer A that waits, earns utility $p_2\,(v_A - p_2)^+$. Therefore, she will purchase in the advance period if $v_A \geq \hat{v}_A$, where

$$\hat{v}_A = \begin{cases} 2\sqrt{p_1}-1 & p_2 < p_1 \leq \frac{1}{4}\,(1+p_2)^2 \\ 2\sqrt{p_1-p_2}+2p_2-1 & \frac{1}{4}\,(1+p_2)^2 < p_1 \leq 1-p_2+p_2^2 \\ 1 & 1-p_2+p_2^2 < p_1 \leq 1. \end{cases}$$

An uninformed buyer A faces the same tradeoff as in the original case. She earns an expected utility of $\mathcal{U}_1 = \frac{7}{12} - p_1$ if she purchases in period 1 and an expected utility of $\mathcal{U}_2 = p_2\int_{p_2}^{1}(v-p_2)\,dv = \frac{1}{2}p_2\,(1-p_2)^2$, if she waits.

Assume first that the firm prices such that the uninformed buyer A purchases in advance, i.e., the firm sets $p_1 \leq 7/12 - p_2\,(1-p_2)^2/2$. In this equilibrium, if there is no purchase in the advance period, then the firm concludes that buyer A was informed, but his value was too low to purchase. That is, the probability that there is no purchase in the advance period is $\beta\hat{v}_A$. The spot period revenue depends on the value of the spot period price, p_2, relative to the threshold $\hat{v}_A$. If $p_2 \geq \hat{v}_A$, then the informed buyer A does not purchase on the spot, the spot period revenue is $\Pi_2 = p_2\,(1-p_2)$ and the optimal spot period price is $1/2$. If $p_2 < \hat{v}_A$, then an informed buyer A may purchase in the spot period. Period 2 revenue is $\Pi_2 = p_2\left(1-\frac{p_2^2}{\hat{v}_A}\right)$, which is maximized when $p_2 = \min\left\{\hat{v}_A, \sqrt{\hat{v}_A/3}\right\}$, which is interior if $\hat{v}_A^r \geq 1/3$. Period 1 revenue is

$$\Pi_1 = (1-\beta\hat{v}_A)\,p_1 + \beta\hat{v}_A\Pi_2^*.$$

If $p_2 \geq \hat{v}_A$, an informed A does not purchase in period 2. This implies that $\hat{v}_A = 2\sqrt{p_1}-1$. Because buyer A doesn't purchase in period 2 (regardless of whether he was informed or not), the spot period revenue is $\Pi_2 = p_2\,(1-p_2)$ and the optimal spot period price is $1/2$. This should be subgame perfect, so the firm always charges $p_2^* = 1/2$. Because in this case, an uninformed buyer A purchases in period 1, we must have that the expected utility in advance is higher than the expected utility from waiting. That is, we must have that $\frac{7}{12} - p_1 \geq \frac{1}{2}p_2\,(1-p_2)^2$, which in this case requires that the firm sets $p_1 \leq 7/12 - 1/16 = 25/48$, which is greater than the subgame perfect second period price ($p_2 = 1/2$). Revenue is:

$$\Pi_1 = \frac{(1-\beta\hat{v}_A)\,(1+\hat{v}_A)^2 + \beta\hat{v}_A}{4} = (1-\beta\hat{v}_A)\,p_1 + \beta\hat{v}_A/4.$$

In the relevant range ($\hat{v}_A \leq 1/2$) the function is increasing in $\hat{v}_A$ (and p_1). Therefore, to maximize revenue, we choose the largest possible p_1 that satisfies the constraints. This results in: $p_1 = \frac{25}{48}$, $p_2 = \frac{1}{2}$, $\hat{v}_A = 2\sqrt{25/48}-1 \approx 0.443$ and revenue of: $\Pi_1^* = \frac{25}{48}\,(1-0.443\beta)+0.11\beta$. This is the same equilibrium as the advance selling equilibrium with resale in the original model. Alternatively, if $p_2 < \hat{v}_A$, an informed A may purchase in period 2. The optimal period 2 price is $p_2\,(\hat{v}_A) = \min\left\{\hat{v}_A, \sqrt{\hat{v}_A/3}\right\}$, which is interior if $\hat{v}_A \geq 1/3$

(it is easy to verify that to find the optimal, it is sufficient to focus on this range). Keeping $\hat{v}_A$ fixed, revenue is increasing in p_1, and $p_1(\hat{v}_A) = (1+\hat{v}_A)^2/4 - \sqrt{\hat{v}_A/3}\left(\hat{v}_A - \sqrt{\hat{v}_A/3}\right)$ is increasing in $\hat{v}_A$. Therefore, we can write Π_1 as a function of $\hat{v}_A$ alone. Plugging in and simplifying, we get:

$$\Pi_1 = \frac{8\sqrt{3\hat{v}_A^3} + 3(1-\beta\hat{v}_A)\left(3 + 10\hat{v}_A - 4\sqrt{3(\hat{v}_A)^3} + 3(\hat{v}_A)^2\right)}{36}.$$

Since $d\Pi_1/d\hat{v}_A > 0$, to maximize revenue we pick the largest $\hat{v}_A$ that satisfies the constraint $\frac{7}{12} - p_1(\hat{v}_A) \geq \frac{1}{2}p_2(\hat{v}_A)(1-p_2(\hat{v}_A))^2$. Plugging $p_1(\hat{v}_A)$ and $p_2(\hat{v}_A)$ and rearranging, we get that the optimal $\hat{v}_A$ is the first root of the quartic equation: $48 - 84\hat{v}_A - 84\hat{v}_A^2 + 32\hat{v}_A^3 + 27\hat{v}_A^4 = 0$, or $\hat{v}_A \approx 0.4286$, $p_1(\hat{v}_A) \approx 0.4911$, $p_2(\hat{v}_A) \approx 0.378$, and $\Pi_1 = 0.4911 - 0.102475\beta$. Comparing the two cases, we get that setting prices such that the informed A does not purchase in period 2 dominates.

Next consider the case in which the firm prices such that the uninformed buyer A waits, i.e., the firm sets $p_1 > 7/12 - p_2(1-p_2)^2/2$. In this case, if there is no purchase in period 1, it implies that either buyer A was uninformed, or she was informed but her value was too low, and she decided to wait. As in the previous case, the period 2 revenue depends on the value of the spot period price relative to the threshold $\hat{v}_A$. If $p_2 \geq \hat{v}_A$, then if the firm has a unit in the spot period, then it can still potentially sell to buyer A only if she was uninformed (with probability $1-\beta$). An informed buyer A in this case does not purchase in period 2 given that $p_2 \geq \hat{v}_A$. Therefore, period 2 revenue is:

$$\Pi_2 = \frac{p_2\left(\beta\hat{v}_A(1-p_2) + (1-\beta)\left(1-p_2^2\right)\right)}{1-\beta(1-\hat{v}_A)},$$

which is maximized when

$$p_2 = \max\left\{\hat{v}_A, \frac{\sqrt{3 - 3\beta(2-\hat{v}_A) + \beta^2(3 - 3\hat{v}_A + \hat{v}_A^2)} - \beta\hat{v}_A}{3(1-\beta)}\right\}.$$

The solution is interior when $\hat{v}_A \leq v'_A$ where

$$v'_A = \frac{\beta + \sqrt{12 - 16\beta + 5\beta^2}}{6 - 2\beta}.$$

Alternatively, if $p_2 < \hat{v}_A$, an informed buyer A may still purchase in period 2, and the firm's revenue is:

$$\Pi_2 = \frac{p_2\left(\beta\hat{v}_A\left(1 - \frac{p_2^2}{\hat{v}_A}\right) + (1-\beta)\left(1-p_2^2\right)\right)}{1-\beta(1-\hat{v}_A)},$$

which is maximized when $p_2 = \min\left\{\hat{v}_A, \sqrt{\frac{1-\beta(1-\hat{v}_A)}{3}}\right\}$. The solution is interior when $\hat{v}_A > v''_A$ where $v''_A = \left(\beta + \sqrt{12(1-\beta) + \beta^2}\right)/6$. Period 1 revenue is

$$\Pi_1 = \beta(1-\hat{v}_A)p_1 + (1-\beta(1-\hat{v}_A))\Pi_2^*.$$

To find the equilibrium, we consider two cases. If $p_2 \geq \hat{v}_A$, an informed A does not purchase in period 2, her utility from waiting is 0, and $\hat{v}_A = 2\sqrt{p_1} - 1$. The optimal period price 2 is

$$p_2(\hat{v}_A) = \max\left\{\hat{v}_A, \frac{\sqrt{3 - 3\beta(2-\hat{v}_A) + \beta^2(3 - 3\hat{v}_A + \hat{v}_A^2)} - \beta\hat{v}_A}{3(1-\beta)}\right\},$$

which is interior if $\hat{v}_A < v'_A$. The firm's total revenues are:

$$\begin{aligned}\Pi_1 &= \beta(1-\hat{v}_A)p_1 + (1-\beta(1-\hat{v}_A))\Pi_2 \\ &= \beta(1-\hat{v}_A)p_1 + \begin{cases}\frac{z\left(\beta(12+\hat{v}_A(z-6))-\beta^2\left(6(1-\hat{v}_A)+\hat{v}_A^2\right)-6\right)}{27(1-\beta)^2} & \hat{v}_A < v'_A \\ \hat{v}_A\left(\beta(1-\hat{v}_A)\hat{v}_A + (1-\beta)\left(1-\hat{v}_A^2\right)\right) & otherwise\end{cases}\end{aligned}$$

where $z = \sqrt{3-3\beta(2-\hat{v}_A)+\beta^2(3-3\hat{v}_A+\hat{v}_A^2)}$. Since $p_1 = (1+\hat{v}_A)^2/4$ is monotone in $\hat{v}_A$, we can substitute p_1 and optimize Π_1 with respect to $\hat{v}_A$ alone. Let Π_1^1 be the first part of the revenue function ($\hat{v}_A < v'_A$) and Π_1^2 be the second part. Analyzing the two parts of the revenue function separately, we get that Π_1^1 is unimodal (strictly concave for $\beta \le \frac{3}{46}\left(3+7\sqrt{3}\right)$ and that Π_1^2 is strictly concave in the range. Let v_A^1 and v_A^2 be the unconstrained optimal thresholds for the two parts of the revenue functions, respectively. There are three candidates for the optimal threshold: v'_A, v_A^1 and v_A^2. Evaluating $\tau_1 = d\Pi_1^1(\hat{v}_A = v'_A)/d\hat{v}_A$ and $\tau_2 = d\Pi_1^2(\hat{v}_A = v'_A)/d\hat{v}_A$, we get that $\tau_i < 0 \ \ \forall i = 1,2$ if $\beta < 2/3$ and $\tau_i \ge 0 \ \ \forall i$, otherwise. This implies that the optimal $\hat{v}_A$ in this case is unique and is given by:

$$\hat{v}_A^* = \begin{cases} v_A^1 & \beta < 2/3 \\ v_A^2 & otherwise,\end{cases}$$

where v_A^1 is the third root of a sextic polynomial (available from the authors) and $v_A^2 = \frac{2\sqrt{6(2-\beta)}+3\beta}{3(4+\beta)}$. Plugging in and evaluating, we verify that the constraints hold.

If $p_2 < \hat{v}_A$, an informed A may purchase in the spot period. The optimal period 2 price is $p_2 = \min\left\{\hat{v}_A, \sqrt{\frac{1-\beta(1-\hat{v}_A)}{3}}\right\}$, which is interior if $\hat{v}_A > v''_A$. As in the previous case, the revenue function can be written as a piece-wise function of $\hat{v}_A$ alone:

$$\Pi_1(\hat{v}_A^r) = \begin{cases}\frac{\beta(1-\hat{v}_A)^3}{4} + \hat{v}_A\left(1-\hat{v}_A^2\right) & \hat{v}_A < v''_A \\ \frac{2}{3}\sqrt{\frac{(1-\beta(1-\hat{v}_A))^3}{3}} + \frac{\beta(1-\hat{v}_A)\left(7-4\beta(1-\hat{v}_A)+6\hat{v}_A+3\hat{v}_A^2-4\hat{v}_A\sqrt{3-3\beta(1-\hat{v}_A)}\right)}{12} & otherwise\end{cases}$$

Let Π_1^3 be the first part of the revenue function ($\hat{v}_A < v''_A$) and Π_1^4 be the second part. Both functions are unimodal. Evaluating $\tau_3 = d\Pi_1^3(\hat{v}_A = v''_A)/d\hat{v}_A$ and $\tau_4 = d\Pi_1^4(\hat{v}_A = v''_A)/d\hat{v}_A$, we get that $\tau_i > 0 \ \ \forall i = 1,2$ and $\forall\beta$. This implies that the optimal $\hat{v}_A$ in this case is unique and is given by the solution to: $d\Pi_1^4/d\hat{v}_A = 0$ (which is a solution to a quintic equation that can be easily evaluated numerically). Plugging $p_1(\hat{v}_A)$ and $p_2(\hat{v}_A)$ in the solution, we verify that the constraints hold. Comparing the two cases, we get that this case dominates the $p_2 \ge \hat{v}_A$ case and therefore the price skimming revenue with forward-looking behavior is greater than the revenue with an informed A leaving the market.

□

Theorem 9 establishes that as in the no recourse mechanism a forward-looking buyer improves the revenue with price skimming, but does not change the advance selling equilibrium.

C.1.3 Overbooking

Only an increased price path may provide benefit to an informed buyer A and hence we focus on it. The advance selling equilibrium is analogous to the one described in Theorem 5. The equilibrium is established in the following Theorem.

Theorem 10. *With the overbooking mechanism, there exists a unique threshold β^b, such that (i) if $\beta \le \beta^b$ the firm implements an advance selling strategy which involves an increasing price path equivalent to the case* *in which the informed buyer A is impatient; (ii) otherwise, the firm implements a price skimming strategy. In that equilibrium,* $p_o^* = (1+b^*)/2$*,* $b^* = p_1^*$*,*

$$p_2^* = \sqrt{\frac{1-\beta\left(1-\tilde{v}_A^*\right)}{3}}$$

and

$$9\beta\mathcal{Y}\tilde{v}_A^{*3} + \left(-10\sqrt{3}\beta + \left(9-3\beta+8\beta^2\right)\mathcal{Y}\right)\tilde{v}_A^{*2}$$
$$= \left(8\sqrt{3} - 10\sqrt{3}\beta - \left(6+3\beta-16\beta^2\right)\mathcal{Y}\right)\tilde{v}_A^* - \left(1-9\beta+8\beta^2\right)\mathcal{Y},$$

where $\mathcal{Y} = \sqrt{1-\beta\left(1-\tilde{v}_A^*\right)}^{-1}$ *and* ω *is given by (1). Revenue and surplus in this equilibrium are equivalent to the revenue and surplus in the price skimming equilibrium with the reselling mechanism.*

Proof. To maximize its profit the firm offers buyer B the unit in period 2 at the price $p_o = (1+b)/2$. In period 1 the informed buyer A's gets utility $\mathcal{I}_1 = v_A p_o + b\left(1-p_o\right) - p_1$ of she buyers and $\mathcal{I}_2 = p_2\left(v_A - p_2\right)^+$, if she waits, where the operator $(x)^+ = \max\{0, x\}$. If $\mathcal{I}_2 = 0$, the case is equivalent to the original model in which the informed buyer leaves the market. Therefore, we focus on the case in which p_2 is chosen such that $\mathcal{I}_2 = p_2\left(v_A - p_2\right)$ for all customers that wait and verify that this is indeed the case in equilibrium. An informed buyer A purchases when $v_A \geq \tilde{v}_A$, where

$$\tilde{v}_A = \frac{2p_1 - b + b^2 - 2p_2^2}{1+b-2p_2}.$$

Given the one-to-one relationship between $\tilde{v}_A$ and p_1, the firm's period 1 decision task to "select p_1 and b" can be reframed as "select $\tilde{v}_A$ and b" and the resulting period 1 price is

$$p_1 = \frac{1}{2}\left(b\left(1-b\right) + \tilde{v}_A\left(1+b\right) - 2p_2\left(\tilde{v}_A - p_2\right)\right).$$

Say there isn't a sale in period 1. In equilibrium either buyer A was uninformed or he was informed but had a value too low to justify purchasing (i.e., $v_A < \tilde{v}_A$). In the first case, there are two buyers in period 2 whereas in the second case only buyer B remains in period 2. Let ω be the probability buyer A is in the market in period 2 (in equilibrium) conditional on there not being a sale in period 1:

$$\omega\left(\tilde{v}_A\right) = \frac{1-\beta}{1-\beta+\beta\tilde{v}_A}$$

The period 2 revenue function is

$$\Pi_2 = \omega p_2\left(1-p_2^2\right) + \left(1-\omega\right)p_2\left(1-p_2^2/\tilde{v}_A\right)$$

Note that conditional on $\tilde{v}_A$, Π_2 does not depend on b. The optimal p_2 is

$$p_2\left(\tilde{v}_A\right) = \sqrt{\frac{1-\beta\left(1-\tilde{v}_A\right)}{3}}$$

Consider the period 1 decisions, $\tilde{v}_A$ and b. The firm's period 1 revenue is

$$\Pi_1\left(b, \tilde{v}_A\right) = \frac{1}{4}\beta\left(1-\tilde{v}_A\right)\left(1-b^2+2\tilde{v}_A\left(1+b\right) - 4p_2^*\left(\tilde{v}_A - p_2^*\right)\right) + \left(1-\beta+\beta\tilde{v}_A\right)\Pi_2$$

The period 1 revenue is maximized with $b = \tilde{v}_A$. The revenue function can therefore be written as

$$\Pi_1\left(\tilde{v}_A\right) = \frac{1}{4}\beta\left(1-\tilde{v}_A\right)\left(\left(1+\tilde{v}_A\right)^2 - 4p_2^*\left(\tilde{v}_A - p_2^*\right)\right) + \left(1-\beta+\beta\tilde{v}_A\right)\Pi_2,$$

which is equivalent to the revenue function with price skimming in the reselling mechanism. Hence, overbooking and price skimming is equivalent (in equilibrium) to reselling and price skimming, i.e., same revenue and social welfare.

□

C.1.4 Refunds

The refund equilibrium with a patient informed buyer A is equivalent to the one with an impatient buyer because an informed buyer does not benefit from a refund policy. If β is low so that an advance selling strategy is optimal, the price path is increasing and an informed buyer A does not wait irrespective of the level of patients. If β is high so that a price skimming strategy is optimal, a refund mechanism is equivalent to a no recourse strategy.

C.2 Random Priority Rule

The main model assumes that if the firm holds the unit in period 2, buyer B gets priority over buyer A. In this section, we generalize the assumption and let the allocation be random. Assume that buyer A gets priority over buyer B with probability ϕ. (That is, the main model is a special case in which $\phi = 0$.) This allocation rule implies that if buyer A waits, his utility is

$$\mathcal{U}_2(\phi) = (\phi + (1-\phi)\,p_2)\int_{p_2}^{1}(v - p_2)\,dv = \frac{1}{2}(\phi + (1-\phi)\,p_2)(1-p_2)^2,$$

where $\mathcal{U}_2$ increases in ϕ. Consider the no recourse case. With an advance selling strategy, the constraint $\mathcal{U}_1 \geq \mathcal{U}_2$ is binding. Therefore, the optimal $p_1^*(\phi)$ deceases in ϕ which implies that the firm's revenue from advance selling decreases with ϕ. With a price skimming strategy, to entice buyer A to wait, the first period price needs to be sufficiently high and the constraint becomes less restrictive (easier to satisfy) as ϕ increases. That is, since the constraint was not binding when $\phi = 0$, it is not binding $\forall \phi \geq 0$, which implies that the revenue function with price skimming does not change with the allocation rule. Similar arguments can be derived for the different recourse strategies. Overall, we conclude that the priority rule assumption does not impact the results qualitatively. Revenues decrease in ϕ with an advance selling strategy and do not change with a price skimming strategy. Consequently, the firm benefits from a small ϕ. Of course, the firm cannot distinguish between customers, so it does not have control over the allocation rule.