Why Facilitate Ticket Exchanges and Other Reselling Markets: Pricing Capacity and Recourse Strategies

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Abstract

Perishable capacity is often sold before it is used (e.g., tickets sold weeks before a sporting event) which creates the opportunity to include in the pricing mechanism a recourse strategy, i.e., allowing the firm or buyer to change ownership after an initial transaction. For example, a buyer could be allowed to resell the purchased unit to another buyer (e.g., a ticket exchange), or the firm could offer the option of a partial refund if the buyer prefers to relinquish it, or the firm could overbook, i.e., sell some of its capacity twice with some compensation to those denied service. Recourse mechanisms are controversial. For example, why should a firm allow its customers to profit from reselling the firm's good, especially if the "customer" is merely a speculator with no actual value for consuming it? We find that allowing such recourse can substantially increase the firm's profit because it allows the firm to ensure more of its capacity is sold and effectively enables the firm to sell that capacity at multiple prices. Among the recourse strategies, reselling is generally best. In fact, reselling is so effective that speculators play no role, and should not be a concern to the firm. We conclude that a firm selling capacity in advance should generally adopt some recourse strategy, and when feasible, reselling is particularly attractive.

1 Introduction

Many firms sell perishable capacity to consumers, capacity that bundles a service with a particular moment in time. Examples include airline flights, hotel rooms, cruise ships, sporting events, music concerts, theatrical events and many others. In these markets consumers learn their preferences over time but are aware of their potential interest in the product well in advance of the moment of delivery: e.g., a person may know in January that she has an interest to take a cruise the second week of July. If the firm sells its capacity in advance, then what options are available to the firm or to consumers if they "change their mind", i.e., what recourse does the firm and/or consumers have if after making the initial commitment (e.g., a cruise ship booking), they want to modify their agreement before the delivery date? We evaluate several options, including the (asymptotically) optimal mechanism.

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In these settings the firm has roughly two approaches for timing when to sell to consumers: (1) sell on the "spot", close to the time of delivery, when consumers have resolved uncertainty over their preferences, or (2) sell in "advance", ahead of the delivery time, when consumer preferences are somewhat uncertain. It has been shown that advance selling can be remarkably effective, despite the fact that the firm needs to offer a price discount to compensate the buyer for their preference uncertainty (Gale & Holmes 1993). Advance selling works because the firm can prefer a sure sale for less in advance over an uncertain sale for more later on.

If advance sales are allowed, then the firm needs to consider what happens if circumstances change (for the consumer or the firm) that may warrant a modification to the initial terms. Rather than ignoring this possibility or leaving these situations to some ad-hoc renegotiation process, the firm and consumers can plan for these contingencies by explicitly including specific policies into the initial agreement, thereby allowing both parties to anticipate correctly the consequences of possible outcomes. We refer to these policies as "recourse strategies".

We evaluate the set of recourse strategies related to who owns the good: refunds, overbooking and reselling. With refunds the initial buyer can return the unit to the firm for a pre-specified partial reimbursement if the consumer learns that their value for the good is relatively low. The firm then attempts to sell the unit to another consumer. With overbooking the firm recognizes that it can sell some of its capacity twice. If in doing so the firm finds itself with excess demand, it offers compensation to early buyers to take back their unit in an effort to close the shortfall. With reselling the firm allows the initial buyer to sell the unit to another consumer through some market setting price mechanism.

Given the breadth of possible recourse strategies, it is useful to define a framework to understand the firm's mechanism design options. At a high level, the firm's first goal is to generate value in the system. There are two means for generating value: (i) transfer ownership of the unit to some consumer because consumers receive some (though, maybe not much) value from the unit whereas the firm surely receives zero value from owning the unit at the end of the horizon; and (ii) conditional that a transfer is made, transfer the unit to the consumer who values it the most. Presuming some value is generated, the second goal of the firm is to extract some of that value for itself, i.e., to earn revenue. Unfortunately, there is tension among these goals, i.e., a selling mechanism cannot simultaneously maximize all forms of value generation and value extraction. For example, Myerson (1981) demonstrates that in a one period model the seller's optimal mechanism deliberately does not maximize the probability of a transaction (i.e., limits value creations) so as to increase the fraction of value the firm can extract - the firm is willing to risk not making a transaction because this increases the share of value the firm can earn if there is a transaction. This issue also arises when capacity is sold over time. To increase the probability of a transaction it is tempting to sell to the first willing buyer, but this may reduce the chance the buyer with the highest value actually uses the product.

The two price strategies without recourse, spot selling and advance selling, take different approaches to value creation and value extraction. Advance selling emphasizes the probability of some transaction at the expense of possibly not allocating the unit to the customer with the highest value. Spot selling increases the probability that the customers with the highest value receives the good, but accepts the risk that a transaction might not occur, thereby leaving the system with a lost opportunity.

Recourse strategies are effective because they can increase the probability transactions are made (relative to spot selling) and the probability the highest value consumers use the product (relative to advance selling). Refunds allow the initial buyers to return the units if the buyers' values turn out to be low, thereby giving the firm the opportunity to possibly sell the returned units to buyers with higher values. With overbooking the firm first determines if high value buyers exists, and if so, then tries to buy back capacity from initial buyers with lower values. Both refunds and overbooking help to improve the allocation of units to the buyers, but neither is perfect. With either mechanism the firm may compensate a customer to accept back their unit and then be unable to sell the unit to another customer, leaving the firm with a loss. Reselling attempts to address the drawbacks of refunds and overbooking by letting the market decide a clearing price to balance supply and demand, thereby avoiding wasted capacity and increasing the odds valuable trades are made.

Among recourse mechanisms, reselling may be the most controversial. From the firm's perspective, it is not clear why the firm would want to relinquish control over pricing. Doing so could allow speculators to enter the market and possibly lead to competition with consumers for some of the firm's inventory in the spot period, either of which may be detrimental to consumer welfare. Nevertheless, online marketplace companies such as Stubhub (owned by eBay), Ticketmaster (owned by Live Nation), RazorGator and many others have made the exchange of tickets through reselling safer and more efficient. The result has been a rapidly growing market which is expected to increase in value (Technavio 2015). As if the mere existence of these markets isn't enough evidence that they provide value, Lewis *et al.* (2019), in an empirical study of season ticket sales for a sports league, finds that reselling increases seller revenue and consumer welfare.

To preview our results, advance selling is most advantageous relative to spot selling when demand is somewhat limited. In those cases it is more important to ensure a transaction is made rather than to identify the buyer with the highest value. But when demand is ample, spot selling is preferred because then it is more important to sell to the highest valued buyers. No matter the level of demand, adding a recourse strategy to advance selling always makes selling in advance the preferred mechanism. This is particularly true when demand is ample because recourse addresses the limitation of advance selling - recourse increases the odds that the buyer with the higher value receives the good. When demand is somewhat limited, the first priority is to ensure capacity is not wasted, so adding a recourse strategy to advance selling is less critical. Among the recourse strategies, reselling performs the best, because it gives some control of pricing to the market, which enables the market to identify value-adding trades. Speculators serve no function, nor are able to profitably participate, in a well designed reselling mechanism - speculators have no value for the good (just like the firm) nor possess better information, so they have no role to play.

2 Related Literature

There is a large literature focused on selling capacity over time. Our work is distinctive in two ways. First, we integrate into a single model many mechanisms that have previously been treated separately. For example, there are models on various recourse mechanisms, but none that compare across mechanisms. Thus, they do not present a theory for the reasons why one mechanism is better than another and under what conditions. Second, we presume efficient markets and rational, forward looking agents. Consequently, our results do not rely on exogenously imposed market frictions such as restrictions on price adjustments or externally imposed costs.

Especially when selling perishable capacity well in advance of its usage, it is possible that consumers are initially somewhat uncertain regarding their preferences. An advance selling strategy, despite this uncertainty, can be highly effective for the firm: Gale & Holmes (1993) show that advance selling allows a monopolist firm to price discriminate between consumers who are relatively indifferent across products (e.g., peak and off-peak flights) and those that have stronger preferences; Dana (1998) shows that advance purchase discounts can arise in a competitive market; DeGraba (1995) demonstrates that a firm can be better off selling a limited amount of capacity in advance to consumers unsure of their preferences; Xie & Shugan (2001) emphasize that advance selling can be effective even with ample capacity; Chu & Zhang (2011) find that it is always in the firm's interest to sell to consumers with less than perfect preference information; advance selling can be used to update a seller's demand forecast (Moe & Fader 2002; Chu & Zhang 2011; Li & Zhang 2013); and Cachon & Feldman (2011) show that advance selling via subscriptions can be effective even in services prone to congestion, despite the limited ability of subscriptions to control congestion. Nevertheless, some limitations of advance selling have been identified: Xie & Shugan (2001) and Prasad *et al.* (2011) show that advance selling is not optimal if marginal costs are high and Cachon & Feldman (2017) show that advance selling can harm firms by increasing the intensity of competition. Here, we find that advance selling provides no benefit to the firm over spot selling when demand is ample relative to capacity. None of these studies on advance selling consider recourse mechanisms.

Among recourse mechanisms, reselling has drawn the most attention. Early work focuses on reselling by individuals who do not value the firm's good, i.e., speculators. These resellers have been generally viewed as undesirable for a firm: Roth (2007) describes reselling by speculators as a repugnant transaction. For example, when late arriving consumers have higher valuations than early consumers, a firm might want to sell with an increasing price path. But Courty (2003) argues that speculators prevent the firm from implementing that strategy because they create competition to sell to the high value consumers. However, he does not consider the possibility of consumer reselling, possibly because at that time technology was not available to support an efficient consumer reselling market. We find that consumer reselling allows the firm to increase its revenue and eliminates speculators.

Several papers document the problem of underpricing (i.e., the firm's tendency to set a price at a level at which demand exceeds supply), which may arise due to the difficulty in pricing hard-to-price goods or the desire to price fairly. Regardless of the reason, pricing below market invites speculation and reduces revenue. Instead of banning resale, which is practically difficult, these papers offer solutions to circumvent entry by speculators and increase revenue. Bhave & Budish (2017) propose running auctions to allow better price discovery and Courty (2019) proposes a full refund for returned units which are then randomly allocated. In contrast to our model, both papers assume that customers are aware of their value for the unit and that the firm does not or cannot set the optimal price in the primary market.

Some work suggests that a firm can benefit from speculators when there are exogenously imposed constraints, such as the firm has restricted control over its pricing: In Su (2010) speculators indirectly allow a firm to implement dynamic pricing when the firm is unable or unwilling to adjust its prices; and in Cui *et al.* (2014) speculators with lower transaction costs serve to transfer units from consumers with low value to consumers with high value. In our model there are no restrictions on what prices can be charged, so speculators play no role (i.e., they are unable to enter and earn a profit). The function of transferring units between consumers with different preferences is important and valuable to the system, but can be accomplished via a reselling market. In general, it is not clear why speculators should play a major role in an efficient market. Like the firm, speculators have zero value for the good, and therefore face a disadvantage in the resale market relative to a seller that does value the good, i.e., speculators are not as willing to pay as much for the good as consumers who have access to reselling. And speculators are likely to have inferior market data relative to the firm for setting appropriate prices. Hence, speculators are more likely to exist in markets with significant trading frictions. The availability of inexpensive information technology has likely reduced these frictions, thereby enabling efficient consumer-to-consumer reselling exchanges (e.g., StubHub).

In our model the firm has full control over reselling. This could be achieved via ownership of the reselling market or through careful contractual control of a third-party. See Zou & Jiang (2020) for a discussion of the impact of an independent reselling market.

As in our model in which consumers arrive sequentially, Yang *et al.* (2017) consider reselling positions in a queue. However, consumers in their model do not learn information over time regarding their valuation and they do not consider dynamic pricing. Nevertheless, in their setting they demonstrate that social welfare and firm profits can increase substantially by allowing consumers to resell.

Some work considers refunds as a recourse mechanism. As in our model, Xie & Gerstner (2007), Gallego & Sahin (2010) and Cui *et al.* (2014) study a monopolist selling to consumers who begin the selling horizon somewhat uncertain of their preferences. With a refund a consumer pays the full price upfront but can receive a partial refund if the consumer later learns of a low value for the good. Equivalently, this can be implemented using options - the consumer pays a non-refundable fee for the option to purchase, and an exercise fee later on if the consumer decides to purchase. It is shown that refunds/options can increase the seller's revenue but they are not compared against alternative recourse mechanisms. Guo (2009) extends Xie & Gerstner (2007) to a competitive setting and demonstrates that refunds may no longer be offered, thereby suggesting that competition is a reason for the limited use of refunds in practice. We offer an alternative explanation for the narrow application - refunds are not the most effective of the recourse mechanisms for the firm.

Overbooking is the practice of selling beyond capacity: e.g., selling more tickets than seats on a flight, or more reservations for a hotel than rooms, or scheduling more appointments in a day than a doctor could actually deliver. Most research focuses on overbooking as a strategy to mitigate the consequences of customers who do not "show up" to actually use the good they purchased: e.g., Weatherford & Bodily (1992), Biyalogorsky *et al.* (1999), Karaesmen & Van Ryzin (2004). In our model overbooking is used by the firm as a tool to find the highest paying customers. Gallego *et al.* (2008) refer to this form of overbooking as a "callable product" and Biyalogorsky *et al.* (1999) call it "overselling". Both demonstrate that it can increase a firm's revenue. The same is true in our model, but reselling is more effective.

Recourse strategies are valuable only if some information changes over time. In our model there are two sources of evolving information. The critical one is that consumers learn information about their preferences over time. For example, after making the initial purchase, a consumer might discover their value for the good is low, or that some other consumer would be willing to pay even more. The second source of information is the amount of demand - for any given posted price there is demand uncertainty. There is a large number of studies, like spot selling, that avoid (or ignore) the first source of uncertainty regarding preferences and focus only on the second (the amount of demand): Aviv & Pazgal (2008), Liu & van Ryzin (2008), Cachon & Swinney (2009). They find that consumers may be willing to pay a premium early on in the selling season to avoid the risk of being unable to make a purchase later on because no more inventory remains. In our model the firm does not use rationing risk to extract additional revenue from customers.

While we study the recourse strategies that apply in our model, there are other recourse strategies implemented in practice. For example, with a price matching guarantee the firm agrees to change the price paid after some information is learned (Lai *et al.* 2010, Huang *et al.* 2017, Pang *et al.* (2021)). In our model there is no need/justification for such price adjustments. Alternatively, there could be a change in the quality of service offered to the customer (Biyalogorsky *et al.* 2005), such as a room or seat upgrade, but we do not include multiple types of products in our model.

3 Model

A firm sells capacity to consumers that is used at a particular point in time, such as admission to an entertainment event, transportation services, or lodging. Consumers can anticipate ahead of that time their need for the capacity, albeit only with precise preferences closer to the time of the event. Consequently, the firm can sell its capacity over time, e.g., well in advance or closer to "on the spot" just before when the capacity is used, among many feasible selling mechanisms.

To be specific, the interactions among the consumers and the firm occur over a two period time horizon. The firm has the capacity to serve q units of demand. The firm's good is provided at the end of the horizon. Period 1 is referred to as the "advance" period and period 2 is the "spot" period. The firm incurs zero marginal cost to deliver its product. Unused capacity at the end of the horizon is wasted because the firm receives no value for unsold capacity. (Equivalently, the salvage value is normalized to zero.)

There are d consumers who arrive in period 1, remain to period 2 and seek one unit of the firm's good. Each consumer's value for the good is independent of the other consumers' values and uniformly distributed on the interval $[0, 1]$.

At the start of period 1 each consumer only knows that their value is on the $[0, 1]$ interval. They also know that everyone observes their own value at the start of period 2. For example, a consumer may know (in advance) that she will have some utility from celebrating a daughter's birthday at a basketball game, but she also knows that she only learns closer to the time of the event exactly how much she values it. (See Papanastasiou & Savva (2017) and Feldman *et al.* (2019) for models in which consumer learning is endogenously determined by the firm's actions rather than, as in our model, an exogenous process.)

In period 1 each consumer can choose to request a purchase or not. If they abstain from or are unable to purchase in period 1, they can purchase in period 2 if they desire and if some capacity is available in that period. The value they receive from the good does not depend on which period they are able to acquire it, i.e., they are fully patient. Consequently, there is no notion that consumers receive any additional value from having a set plan well in advance of consumption.

In this market, consumers are strategic in the sense that they correctly anticipate future actions and prices. Consumers are also optimistic (and all are aware of this), in the sense that they believe in period 1 they will be able to obtain a unit in period 2 provided that the firm has units to sell and that they are willing to pay the period 2 price. This optimism results in lower revenue, because the firm is unable to use rationing risk to its advantage–it has been shown that the possibility a consumer may not receive the good works to the advantage of the firm, especially if the firm were able to credibly hold back capacity (DeGraba (1995)).

The parameters and sequence of events are common knowledge to the consumers and the firm. All agents are risk-neutral utility maximizers. The firm's objective is to design the terms of trade to maximize expected revenue (which is equivalent to expected profit given the zero marginal cost for delivering capacity). Even though the firm is not required to sell all of its capacity, the firm could do so if the firm were to give away its product for free, i.e., $q < d$. For notational convenience, let ϕ be the ratio of capacity to demand, $\phi = q/d$.

We evaluate two versions of the model. In the "large" market there are many consumers, so each consumer's potential demand is a trivially small amount of capacity. In the "small" market there is a finite number of consumers. In both markets the firm does not know an individual consumer's preference. However, in the large market there is no aggregate uncertainty regarding realized demand, whereas such uncertainty exists in the small market.

4 Large Market

The large market is the asymptotic limit of a series of markets that scale capacity and demand proportionally. In particular, in the large market d is the size of a mass of infinitesimally small consumers and q is the amount of capacity that can be served relative to potential demand. In such market there is no aggregate uncertainty. For example, it is known in advance (period 1) the fraction of consumers that have a certain value or greater, but it is not known in advance which ones do. Three mechanisms are considered: the basic spot selling only in period 2, advance selling (without recourse), and the optimal mechanism. See the Appendix for additional details beyond those provided in this section.

4.1 Spot Selling

With spot selling the firm does not make the product available in period 1 when consumers are unsure of their value. (Equivalently, the firm sets a sufficiently high period 1 price to eliminate advance sales.) Instead, the firm sells exclusively in period 2 when all consumers know their value for the good. This ensures that the firm sells only to the consumers with the highest value for the good, but it does not assure full utilization of capacity.

Table 1. Spot Selling Outcomes ($\phi = q/d$)

	q_s^*	p_s^*	R_s^*
$\phi \leq \frac{1}{2}$	q	$1-\phi$	$(1-\phi)\,q$
$\frac{1}{2} < \phi < 1$	$\frac{q}{2\phi}$	$\frac{1}{2}$	$\frac{1}{4\phi}q$

Table 2. Advance selling ($\phi = q/d$)

	q_1^*	p_1^*	p_2^*	R_a^*
$\phi \leq \frac{1}{3}$	0	–	$1-\phi$	$(1-\phi)\,q$
$\frac{1}{3} < \phi < 1$	$\left(\frac{1}{1+\phi}\right)\left(3-\frac{1}{\phi}\right)q$	$\frac{1}{8}(3-\phi)(1+\phi)$	$\frac{1+\phi}{2}$	$\frac{1}{8\phi}(1+\phi)^2\,q$

Let $R_s(q,d,p_s)$ be the firm's revenue when it sells q units to d customers in period 2 at price p_s,

$$R_s(q,d,p_s) = \min\{(1-p_s)\,d, q\}\,p_s$$

Table 1 summarizes the main results under the firm's optimal spot selling mechanism, including the quantity sold, q_s^*, the price, p_s^*, and the firm's revenue, R_s^*. When demand is ample, $\phi \leq 1/2$, then the firm sells its entire capacity. However, when demand is limited, $\phi > 1/2$, the firm prefers to only sell a portion of its capacity.

4.2 Advance selling without recourse

The firm can sell units in period 1 (in "advance") without recourse, i.e., all period 1 buyers who make a purchase consume the good even if they observe in period 2 a relatively low value. Consumers attempt to purchase in period 1 only if they believe doing so is preferable over the option to wait to period 2. If they were to wait, then they could make the purchase decision after observing their value for the good. Consumers recognize that the period 2 price is likely to be different than the period 1 price.

The firm's revenue is

$$R_a = p_1 q_1 + R_s(q-q_1, d-q_1, p_s^*),$$

where p_1 is a price consumers are willing to pay to purchase in period 1, q_1 is the number of units the firm chooses to sell in period 1, $q_1 \leq q$, and $R_s(q-q_1, d-q_1, p_s^*)$ is the firm's maximum revenue in the spot period when it sells its remaining $q-q_1$ units to $d-q_1$ customers at the spot period revenue maximizing price, p_s^*. Table 2 summarizes the choices and outcomes in this mechanism.

The firm sells all of its capacity, but when demand is ample, $\phi \leq 1/3$, it does so only in period 2, effectively resorting to spot selling. Otherwise, it sells a portion (but never all) of its capacity in the advance period. In general, the amount sold in advance is decreasing in the amount of demand.

The tradeoff with advance selling is simple - sales are boosted by selling in advance, but advance selling does not ensure capacity is sold to the consumers with the highest values for the good. Increasing sales is most valuable when demand is limited, when some capacity could otherwise go unutilized. However, doing this requires selling at a discount. If demand were ample, the firm would prefer to set a higher price and sell only to those customers who are willing to pay it. Put another way, rather than stating advance selling works *even when* demand is limited (as suggested by Xie & Shugan (2001)), it is better to state that advance selling is useful *only if* demand is limited.

4.3 Optimal Mechanism

The firm's optimal selling mechanism can be implemented as a truth-inducing mechanism (Myerson, 1981): outcomes and transfer payments are based on consumer reports of their private information (i.e., their value for the good) and truthful reporting is the consumers' equilibrium report.

Theorem 1. *The firm's optimal mechanism assigns ownership to all q units available to the buyers in period 1 at price p_1^*. (As $q < d$, q buyers are randomly selected to receive the good in period 1.) In period 2 all buyers with a unit and reported type $p_2^* - t^*$ or lower relinquish their unit and receive $p_2^* - t^*$ in compensation, and all buyers with reported type p_2^* or higher receive a unit, paying p_2^* ($\phi = q/d$):*

$$t^* = \phi$$

$$p_1^* = 1 - 2\phi + \frac{5}{2}\phi^2 - \phi^3$$

and

$$p_2^* = 1 - \phi + \phi^2.$$

The firm's total revenue is

$$R^* = \frac{1}{2}\left(1 + (1-\phi)^2\right) q.$$

Figure 1 displays the firm's profit with each of the three mechanisms and across a range of demands relative to capacity. When demand is limited, spot selling performs poorly, advance selling is able to capture most of the firm's potential, but the optimal mechanism always improves upon advance selling. As demand increases, all mechanisms increase their profit per unit of capacity, as expected, but advance selling begins to lose its relative advantage. When potential demand is three times capacity, $d/q = 3$, advance selling is no longer used. Nevertheless, the optimal mechanism continues to perform better than spot selling, albeit with a smaller advantage.

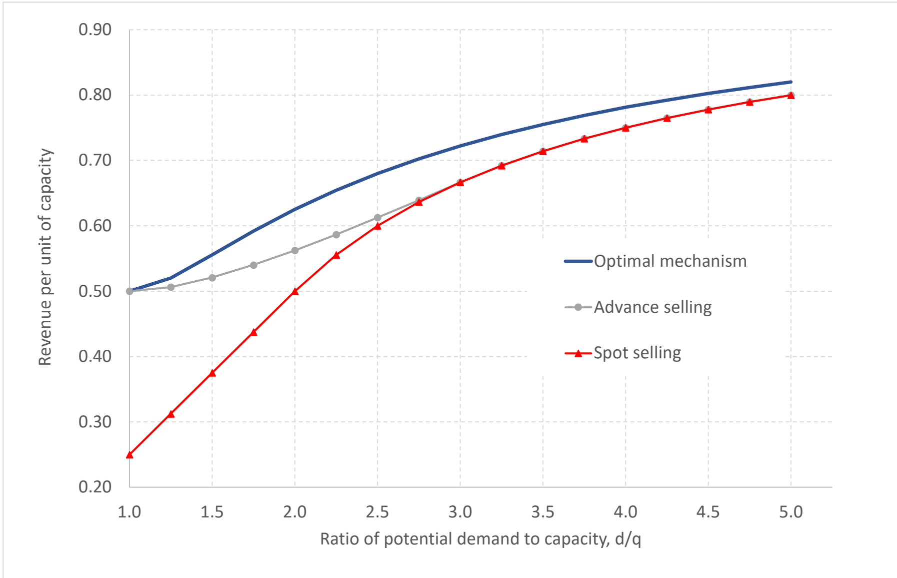

Figure 1. Large Market Firm Revenue

Recall, with advance selling the firm always holds back at least some capacity to sell in period 2. The optimal mechanism does not need to do this because it retains the advantage of advance selling (avoiding wasted capacity) while it also overcomes the limitation of advance selling (possibly selling to the wrong consumer) by enabling units sold in period 1 to consumers with low values to be transferred to consumers in period 2 with higher values.

To understand how the optimal mechanism improves upon spot selling, consider the case with ample demand, $d/q > 2$ (i.e., $\phi < 1/2$), which is most favorable to spot selling. Consequently, with spot selling the firm sells all q units at the highest possible price. This maximizes total welfare because there are no remaining profitable trades - the q consumers with the highest value all receive a unit. It would seem that this is the best the firm could do, yet the optimal mechanism does strictly better. A comparison of the prices begins to explain why:

$$p_1^* < p_s^* < p_1^* + t^*.$$

Like spot selling, with the optimal mechanism all units are sold. However, with the units initially purchased and not transferred to another consumer, the firm earns only p_1^* per unit, which is less than if those units were sold with exclusively a spot market. But with the units initially purchased and transferred, the firm earns $p_1^* + t^*$, which is more than the spot price. Hence, with the optimal mechanism the firm is able effectively to sell all q units at two prices rather than the single price which would occur with spot selling. This allows the firm to price discriminate across the consumers and net a higher revenue. However, total welfare is not maximized because there are some feasible trades that could occur in period 2 between those who obtained a unit in period 1 and received a relatively low value (between p_1^* and p_s^*) and those that were excluded in period 1 yet received a relatively high value (between p_s^* and p_2^*). As seen in other settings, with its optimal mechanism the firm sacrifices some total welfare to increase its share of the (less than optimal) welfare.

Although the firm's revenue varies across the units it sells, the optimal mechanism is not an example of dynamic pricing. In a large market all agents are able to anticipate correctly future prices and availability. While individual consumers learn information (their value for the good), there is no notion that the firm or the market learns information because the consumers are all small. In other words, the period 2 price is not different because it is in response to new information. Instead, it is different so that the firm can earn revenue (indirectly) from high value consumers at a higher price. Even though there is a single segment of consumers based on their valuation for the good, the firm, in effect, creates two segments through the timing of when they purchase.

While the optimal mechanism presumes consumers report their type truthfully, such a mechanism is unlikely to be implemented in practice. Instead, the firm could implement one of several recourse mechanisms. For example, to replicate the optimal mechanism, the firm could sell options in period 1 in which a consumer pays $p_1^* - p_2^* + t^*$ for the option to purchase the unit in period 2 for $p_2^* - t^*$: if the consumer purchases an option and observes a value greater than $p_2^* - t$ in period 2, then the consumer exercises the option and pays a total of p_1^*, whereas if the consumer observes a value less than $p_2^* - t^*$ then the consumer's net utility is only the cost of the option, $p_1^* - p_2^* + t^*$. Another approach charges consumers p_1^* per unit in period 1 while granting them the right to return the unit at the start of period 2 for a refund of $p_2^* - t^*$. The outcomes across the option and refund mechanisms at the end of period 2 are the same for all agents, but the timing of payments is different.

Alternatively, the firm could implement the optimal mechanism via overbooking: q units are sold in period 1 for p_1^*; the firm sells in period 2 at price p_2^* to the remaining $d - q$ consumers; and to satisfy the demand of those period 2 sales, the firm repossesses units from the consumers who purchased in period 1. For overbooking to replicate the optimal mechanism, the period 1 consumers who are denied service must be the $(1 - p_2^*)(d - q)$ consumers with the lowest values. If overbooking is unable to perfectly sort the period 1 consumers by their values, then overbooking adds an additional source of waste to the system, thereby falling short of the optimal mechanism. In practice, it is doubtful such perfect sorting could occur, but it may be sufficiently good to allow overbooking to nearly replicate the optimal mechanism.

The optimal mechanism can also be implemented as a reselling market, such as a ticket exchange. The firm sells its units in advance for p_1^* and then the reselling market establishes a clearing price, p_2^*, to maximize the number of trades given a transfer fee, t^*. It is important for the reselling market to be efficient (i.e., maximize the number of trades), otherwise it too falls short of replicating the optimal mechanism. However, the presence of a transaction fee is not an indication of an inefficient market. Instead, it is a critical source of revenue for the firm. Consequently, it is important for the firm to have control over the exchange, either through ownership or through well specified contracts with a third party.

The potential activity of speculators raises one concern with the implementation of a reselling market. Speculators are consumers who know they have zero value for the good yet nevertheless trade with the goal to earn income. It has been suggested that speculators provide a useful function for the firm if the market is inefficient (Su 2010,Cui *et al.* 2014). However, speculators are unable to participate (profitably) and have no useful role to play in an efficient reselling market. To explain, speculators are at a disadvantage relative to the other consumers because they are not willing to pay the firm's initial price, p_1^*. Only consumers that know they will receive some value for the good are willing to pay p_1^*. Thus, the firm need not be concerned with the possibility that speculators may influence the market and the firm surely does not need speculators to maximize its revenue.

Another concern with reselling markets is the potential to create competition between the firm and its previous customers. This is not an issue. It is optimal for the firm to maximize (not minimize) the number of consumers who purchase in period 1. To explain, each unit sold in period 1 creates a potential future competitor, which lowers p_2. This logic is intuitive, but incomplete. Selling a unit in period 1 also reduces the supply of units the firm has to sell in period 2 and it reduces demand in period 2. Both of these effects raise p_2. The net effect is that each unit sold in period 1 reduces period 2 supply and demand, but supply falls more than demand, raising the period 2 price. Put another way, with each unit of capacity, the firm prefers that a consumer tries to sell it rather than the firm being forced to try to sell the unit itself. The consumer has some value for the good, whereas the firm does not. Hence, the firm is better off competing against some consumer who is only willing to accept a sufficiently high price than competing against an agent that will take anything for the unit it does not value at all. In sum, the competition the firm should fear is the future competition from itself (who is desperate to unload the units) rather than from consumers (who are not under the same level of pressure to sell).

The transfer fee is critical for the success of the reselling mechanism. Without it, the firm can do no better than spot selling. In fact, with $t = 0$, the period 2 market is exactly like spot selling in the sense that it balances the number of consumers who want to purchase (those with values p_2 and higher) and the supply that wants to sell (those with values p_2 or lower). In period 1 the consumers can anticipate this, so the most they will pay to own a unit is the same price they expect to be able to trade in period 2, i.e. $p_1 = p_2$. If demand is ample, $\phi < 1/2$, then reselling replicates spot selling - with either mechanism the firm sells all of its capacity at the clearing price p_2. But if demand is more limited, $\phi < 1/2$, then reselling without a transfer fee does even worse than spot selling because the firm can remove some supply from the market with spot selling whereas with reselling it cannot prevent the period 1 customers from providing that supply.

Although reselling requires the firm to have access to some transfer fee, if for some reason the firm must charge less than the optimal transfer fee, it may still earn a substantial portion of its optimal revenue: if the transfer fee is $3/4^{th}$ of the optimal, then revenue is still about 92.7% of optimal. In other words, the firm's revenue function is relatively flat about the optimal transfer fee. Similarly, if the optimal transfer fee is imposed on the market but the firm collects only half of it, then the firm's revenue is still at least 89.7% of optimal.

The transfer fee is valuable to the firm with reselling because it motivates period 1 consumers to purchase early (to avoid the higher period 2 price) and allows the firm to effectively collect revenue twice on a single unit of capacity (p_1^* in period 1 and t^* in period 2 if it is transferred). The other recourse mechanisms rely on the same "wedge" between the prices across the periods, but create that wedge with a different set of payments. With refunds and overbooking, the firm controls the period 2 supply (i.e., the number of period 1 units that return to the market) via the amount it pays to accept returns or the amount it pays when it denies service. The transfer fee imposes an explicit cost on period 1 buyers who observe a low value, whereas refunds and overbooking impose an implicit cost. In sum, with all of the mechanisms, the period 1 consumers who relinquish their units experience a net loss due to a partial refund, or incomplete compensation for denial of service, or a transfer fee, but the net loss is less than it would otherwise have been without the recourse.

5 Small Market

In the "large" market (Section 3) there is no aggregate uncertainty because there are many small consumers. This section considers a "small" market in which each consumer is sufficiently large that their presence does influence market outcomes. Consequently, the demand for the firm's product is uncertain: at a price that would certainly clear the firm's capacity in the large market, there can be either excess demand or insufficient demand leading to idle capacity in the small market. Both situations can destroy system value. With excess demand it is possible that some consumers with high value are prevented from participating in the market while consumers with lower values for the good receive the service. Alternatively, idle capacity wastes the opportunity for some consumers to value it, even if just a little bit.

As in the large market, the firm has q units to sell and there are d consumers. However, in the small market integer constraints apply, meaning that each consumer desires precisely one unit of capacity. In all other ways the small market operates like the large market.

We begin with the optimal mechanism for the small market. We then consider each of the mechanisms identified in the large market: spot selling, advance selling (with no recourse), refunds/options, overbooking and reselling in a market exchange. While in the large market, the optimal mechanism can be implemented by all recourse strategies, this is not the case in the small market. Due to demand uncertainty, the distinctions across these mechanisms is more significant in the small market, revealing important differences in implementation and performance. In particular, with refunds consumers face a fixed price to sell their unit but a variable price to buy it. With overbooking consumers face a fixed price to buy units but a variable price to sell it back to the firm. In a reselling market both the buy and sell prices are variable. Additional details on the evaluation of each mechanism are provided in the Appendix.

5.1 Optimal Mechanism

The firm's optimal mechanism in the small market transfers some units to customers in period 1. In period 2 the mechanism allows for recourse, i.e., the exchange of units amongst the firm, the "sellers" (customers who received a unit in period 1) and the "buyers" (customers who do not have a unit at the start of period 2). The period 2 exchange can be implemented using a particular double auction, as described in Theorem 2.

Theorem 2. *The firm's optimal mechanism in the small market (i) selects q_1 customers in period 1 (randomly among the q customers) to receive a unit, each paying p_1^* such that they are indifferent between owning a unit or not, and (ii) implements a double auction in period 2 between customers who purchased in period 1 ("sellers") and customers who did not purchase in period 1 ("buyers"). In the auction sellers bid their value for the good. A buyer with value v_b bids $b(v_b)$,*

$$b(v_b) = \frac{(2(d - q_1) + q_1) v_b - d}{d - q_1} \leq v_b.$$

Let b^i denote the value of the i^{th} highest bid. Define y to equal the smaller of q or the rank of the number of positive bids. Sellers with bids b^y or lower relinquish their unit and receive compensation b^y. Buyers with bids b^y or greater are given a unit and are charged $v_k\left(b^{y+1}\right)$, where

$$v_k(x) = \frac{(d - q_1) x + d}{2(d - q_1) + q_1}.$$

There are no transfers for sellers who retain a unit or for buyers who do not receive a unit.

Theorem 2 does not identify the optimal number of initial units to sell, q_1. A search over possible q_1 leads to the optimal first period quantity and total expected revenue. As with the other mechanisms considered, $q_1 \leq q$ is presumed.[1] We find in our numerical study that, as in the large market, it is optimal to sell all

[1]With $q_1 \leq q$ the firm can sell more than capacity in period 2 but not in period 1. The optimal mechanism readily extends

units in period 1 (i.e., $q_1^* = q$).

In contrast to the optimal mechanism in the large market, in the small market it cannot be implemented with refunds or overbooking. The optimal mechanism does resemble what might be expected from a reselling market: buyers (customers without units) and sellers (customers allocated a unit in period 1) submit bids, units are allocated based on the bids, and customers with higher values are more likely to obtain units. However, this mechanism is not efficient. For example, because buyers bid less than their value, it is possible that the q customers with the highest values do not receive a unit. The mechanism sacrifices some efficiency in the period 2 reselling market so that it can increase the firm's revenue in period 1. In particular, in period 1 a customer is willing to pay the gain in utility from owning a unit. Owning becomes more valuable as the reselling market assigns more priority to sellers, which is done with a bidding process that leaves some buyers without a unit even though assigning them a unit would be welfare enhancing. In fact, the advantage given to sellers may result in a higher sell price than buy price and consequently negative period 2 revenue for the firm.

Although the optimal mechanism can be implemented as a truth inducing mechanism (i.e., each customer's optimal action is to report their value), this is unlikely to be actually put into practice because it would require participants to report their value for the good without a clear understanding of what they will be paid or pay for the unit. A double auction is more realistic to implement in practice, but it is not a simple mechanism for participants. While sellers can bid their value, buyers must be strategic in their bidding (i.e., bid less than their value) and doing so correctly requires that they are aware of the number of initial units sold, q_1, as well as the size of the market, d, and how to use those data to generate a bid. Furthermore, revenue that the firm collects in the resale market depends on the individual bids in a way that cannot be specified as a fixed percentage of prices. And, as already mentioned, the firm's revenue in the resale market can even be negative, which is unusual.

5.2 Spot Selling

Let $S(q_s, d_s, p_s)$ and $R_s(q_s, d_s, p_s) = p_s S(q_s, d_s, p_s)$ be the firm's sales and revenue, when it has q_s units to sell to d_s customers in a single period and it charges price p_s. We show that the revenue function is unimodal, which implies that there is a unique optimal spot selling price, $p_s^*(q_s, d_s)$, when the firm sells q_s units to d_s customers. The resulting firm's optimal spot selling revenue is $R_s^*(q_s, d_s) = R_s(q_s, d_s, p_s^*(q_s, d_s))$.

to the case of $q_1 > q$, which would imply selling more than capacity even in the advance period. Our numerical analysis suggests there is a limited revenue gain from doing this extreme form of overbooking, especially when the number of units sold is reasonably large (e.g., $q = 64$).

5.3 Advance Selling without Recourse

With advance selling the firm can sell some (or all) of its units in period 1 to buyers who do not yet know with certainty their value for the good. If q_1 units are sold in period 1, then the firm can sell $q - q_1$ units in period 2 to the remaining $d - q_1$ customers. The anticipated price in period 2 is therefore the spot period price with the corresponding supply and demand remaining in that period, i.e., $p_s^*(q - q_1, d - q_1)$.

Let $\omega(p_2)$ be a buyer's expected earnings in period 2, if the buyer does not make a purchase in period 1, where the anticipated period 2 price is p_2,

$$\omega(p_2) = \int_{p_2}^{1} (x - p_2)\, dx = \frac{1}{2}(1 - p_2)^2.$$

To explain, a buyer is optimistic and therefore anticipates that they can obtain a unit if they are willing to pay the expected price, p_2. In period 1 a buyer is willing to pay as much as $p_1(q_1)$ for a unit:

$$p_1(q_1) = \frac{1}{2} - \omega(p_s^*(q - q_1, d - q_1)).$$

The firm's revenue is

$$R_a(q_1) = p_1(q_1)\, q_1 + R_s^*(q - q_1, d - q_1).$$

A search over the feasible space, $q_1 \in [0, q]$ yields the optimal quantity to advance sell in period 1 (with $q_1 = 0$ resulting in spot selling).

5.4 Refunds/Options

As in the large market, for any mechanism in which options are sold there exists an equivalent mechanism in which the firm offers refunds in period 1 for a set buyback price. The refund mechanism is described here. Specifically, the firm announces a refund price in period 1 and accepts all returns for that price at the start of period 2. In period 2 the firm can try to sell all returned units along with the units not sold in period 1. Consequently, the period 2 price is variable due to the uncertain number of units that are returned.

Let r be the refund price the firm chooses, where $r = 0$ implies returns are not accepted and results in advance selling. A consumer returns their unit at the start of period 2 for a refund if they observe a value of r or lower. Let $q_r(r)$ be the (stochastic) number of units returned. The firm's available supply in period 2 is $q_2(r) = q - q_1 + q_r(r)$. There are $d - q_1$ customers to sell to in that period and the firm selects an optimal price, $p_s^*(q_2(r), d - q_1)$, for each possible $q_2(r)$, where $p_s^*(0, d - q_1) = 1$.

A buyer in period 1 considers the earnings from waiting to buy in period 2 given that the period 2 price is stochastic. A buyer's expected earning from the choice to not purchase in period 1 (which primarily depends on the period 2 price) is $\pi^b(q_1) = \mathbb{E}\left[\omega\left(p_s^*\left(q_2(r), d - q_1\right)\right)\right]$. A buyer's expected earnings from the option to sell their unit back to the firm in period 2 (which primarily depends on the return price the firm offers) is

$$\pi^a(q_1, r) = \int_0^r (r - x)\, dx = \frac{r^2}{2}.$$

The most the firm can charge in period 1 is the difference in a consumer's earnings between purchasing in period 1 (not including the cost of the purchase) and waiting:

$$\begin{aligned} p_1(q_1, r) &= \frac{1}{2} + \pi^a(q_1, r) - \pi^b(q_1, r) \\ &= \frac{1}{2}\left(1 + r^2\right) - \mathbb{E}\left[\omega\left(p_s^*\left(q_2(r), d - q_1\right)\right)\right]. \end{aligned}$$

The firm's expected revenue is the combination of sales across the two periods, minus the fees paid for returned units,

$$R_r(q_1, r) = p_1(q_1, r)\, q_1 + \mathbb{E}\left[R_s^*\left(q_2(r), d - q_1\right) - q_r(r)\, r\right].$$

A search of the parameter space finds the optimal refund mechanism.

The refund mechanism provides consumers with some insurance. If they purchase in advance but later observe their value to be lower than the refund rate, then they can return the unit to the firm to mitigate their losses. Naturally, the firm can generate revenue by selling this insurance. However, the firm is left trying to sell returned units to a smaller base of consumers in period 2, reducing some of the benefit of selling in advance, and leaving the firm exposed to the risk of idle capacity.

5.5 Overbooking

With the overbooking mechanism the firm sells q_1 units in period 1 for p_1 and then offers the remaining units in period 2 for p_2. The distinguishing feature of this mechanism is that the period 1 purchasing consumers have the right, but not the obligation, to return their unit to the firm in period 2. In particular, if the firm's demand in period 2 exceeds its available supply then the firm can attempt to acquire units from the period 1 consumers to help satisfy the shortfall in demand. Let r_o be the price the firm offers in period 2 to the consumers holding a unit. This price is chosen after observing period 2 demand at the chosen p_2 price. The firm does not know the period 1 consumers' values, so the number of consumers willing to sell their unit at r_o is uncertain, leaving the firm with both the risk of excess supply and inadequate supply. If an insufficient number of units are returned in period 2, then the firm forgoes a sufficient number of period 2 sales so that final demand and supply match.[2]

As is the common practice in the airline industry, consumers here voluntarily relinquish their units and it is the consumers with the lowest values that do so. Furthermore, the airline can adjust their offer to the consumers depending on the realized demand.

Overbooking is analogous to refunds, just in reverse. With refunds the period 1 consumers receive a fixed price to sell their units back to the firm, but the period 2 price to buy is stochastic. With overbooking the period 2 price to buy is fixed, but the price to return units back to the firm is stochastic.

In period 2 there are two decisions, the initial period 2 price, p_2, and then the return price offer after period 2 demand is observed, r_o. In contrast to the refund mechanism for which advance selling was a special case (a firm that sets $r = 0$ implements advance selling), with overbooking the buyback prices are chosen dynamically once demand is observed, and therefore the strategy does not mimic advance selling.

The firm has $q - q_1$ units in period 2 and there are $d - q_1$ customers. Let d_2 be realized demand in period 2 and let d_o be overbook demand in period 2, $d_o = (d_2 - q + q_1)^+$. Let $\pi_o(q_1, p_2, d_o, r_o)$ be the firm's profit from overbook sales. Unlike with spot selling in which the firm has a fixed supply and uncertain demand, here, the firm's overbook demand is known and it is the supply that is uncertain. Let $r_o^*(q_1, d_o)$ be the firm's optimal period 2 rate and let $\pi_o^*(q_1, p_2, d_o)$ be the firm's optimal profit from overbook sales.

The firm's period 2 revenue is

$$R_2(q_1, p_2) = p_2 S(q - q_1, d - q_1, p_2) + \mathbb{E}\left[\pi_o^*(q_1, p_2, d_o)\right].$$

The first term is the expected revenue from sales of the firm's $q - q_1$ units and the second terms is the profit from overbook sales. Let $p_2^*(q_1)$ be the firm's optimal period 2 price given the number of units sold in period 1, q_1, and $R_2^*(q_1) = R_2(q_1, p_2^*)$ the corresponding optimal period 2 revenue.

In period 1 the consumer can either choose to purchase at the offered price, p_1, or wait to period 2 to try to purchase. The most the firm can charge in period 1 is $p_1(q_1) = 1/2 + \pi^a(q_1) - \pi^b(q_1)$, where $\pi^a(q_1)$ is a customer's expected earning from the option to sell their unit back to the firm in period 2,

$$\pi^a(q_1) = \frac{1}{2}\mathbb{E}_{d_o}\left[\left(r_o^*(q_1, d_o)\right)^2\right]$$

[2]In practice a firm might incur an additional penalty for denying service to a period 2 customer. Such a penalty would lower the effectiveness of overbooking.

and $\pi^b(q_1)$ is the buyer's expected earnings from the option to sell their unit back to the firm in period 2,

$$\pi^b(q_1) = \begin{cases} \omega\left(p_2^*(q)\right)\mathbb{E}_{d_o}\left[\left(1-\left(1-r_o^*(q_1,d_o)\right)^q\right)\right] & q_1 = q \\ \omega\left(p_2^*(q_1)\right) & q_1 < q. \end{cases}$$

To explain, if the number of units sold in period 1, q_1, is less than total capacity q, then the buyer expects to have a unit available. However, if the firm sells all q units in period 1, the buyer will only obtain a unit if there are returns (and the probability of returns depends on the realization of d_o).

The firm's total revenue is

$$R_1(q_1) = p_1(q_1)\,q_1 + R_2^*(q_1)\,.$$

The optimal q_1 is found by searching over the feasible space $q_1 \in [0, q]$.

5.6 Reselling

In the reselling mechanism the firm sells up to q_1 units in period 1 for price p_1. In period 2 the remaining $d - q_1$ buyers desire a unit. The supply of units in period 2 include the $q - q_1$ units from the firm and the q_1 units from the period 1 buyers. The firm establishes a reselling market that allows the consumers who purchase in period 1 to sell their unit in period 2. There are many plausible designs for the reselling market. The double auction implementation of the optimal mechanism is one, albeit, as discussed, a complex mechanism. Here we consider a simpler mechanism inspired by the optimal mechanism in the large market.

In this reselling mechanism the firm sells all units in period 1, and for each unit traded in the reselling market in period 2 the firm charges t percent of the price the sellers receive. The primary complication in the design of the market mechanism is to determine the number of trades and trading prices. As is well known (Myerson & Satterthwaite (1983)) the design of a bilateral trading market must sacrifice at least one desirable property. We implement a mechanism developed in McAfee (1992) that is generally efficient (i.e., all welfare enhancing trades are made), budget balancing (i.e., there is a single clearing price) and simple to implement for the participants (i.e., reporting their value is a dominant strategy). In particular, let v_i be the i^{th} largest value of the potential buyers and let w_i be the i^{th} smallest value of the potential sellers. Let y be the maximum number of feasible trades, i.e., y is the largest integer value such that $v_y \geq w_y/(1-t)$, where $y = 0$ is possible. Let

$$p_2 = \frac{1}{2}\left(v_{y+1} + \frac{w_{y+1}}{(1-t)}\right).$$

If p_2 falls in the feasible range for y trades, i.e.,

$$\frac{w_y}{(1-t)} \leq p_2 \leq v_y,$$

then y trades occur at price p_2. Otherwise, $y-1$ trades occur, where the sellers receive $w_y/(1-t)$ and the buyers pay v_y. In either situation the firm collects a transfer fee of t percent from each seller. In the second case with only $y-1$ trades, the firm also collects the difference between the buy and sell prices. In the first case the mechanism satisfies the desirable properties that there is a single clearing price, it is efficient (i.e., maximizes the number of feasible trades) and the agents play dominant strategies. The second case retains the dominant strategies, but forgoes one feasible trade and requires the firm to take an additional spread on each trade. Although the second case is not as desirable as the first, it becomes less likely as the market thickens (i.e., as there are more buyers and sellers).

A closed-form evaluation of this reselling market is complex. Nevertheless, the performance of this market can be readily estimated via simulation. The optimal transfer fee from the large market could be implemented, but a grid search over possible transfer fees can be undertaken to improve the mechanism further.

5.7 Comparison of mechanisms

Figures 2, 3, and 4 display the firm's revenue per unit of capacity across various mechanisms, demand-to-capacity ratios (d/q), and capacities, $q \in \{4, 16, 64\}$, respectively. Revenue per unit of capacity is used to facilitate direct comparisons across capacities that would otherwise vary considerably in total revenue. As in the large market, the demand-to-capacity ratio (the inverse of ϕ) influences the tradeoff between consumer selection (finding the consumers with the highest values) and capacity utilization (i.e., avoiding idle/wasted capacity). When d/q is large, e.g., $d/q > 2$, then proper consumer selection is more important because the sheer number of customers provides assurance that capacity will be utilized. When d/q is more limited, e.g., $d/q < 2$, capacity utilization is more of the focus because the threat of wasted capacity looms large. The variation in the amount of capacity, $q \in \{4, 16, 64\}$ influences how "big" the market is - with $q = 4$ each unit is a substantial portion of total capacity or demand (i.e., the market is indeed "small"), whereas with $q = 64$ the impact of any one unit is muted (i.e., the market is relatively "larger").

The figures reveal that in the small market (i) the recourse mechanisms perform substantially better than spot selling, especially when demand does not exceed capacity by a large margin, (ii) the recourse mechanisms each can perform significantly better than simple advance selling without recourse, (iii) there are considerable differences across the recourse mechanisms, and (iv) reselling is the best recourse mechanism

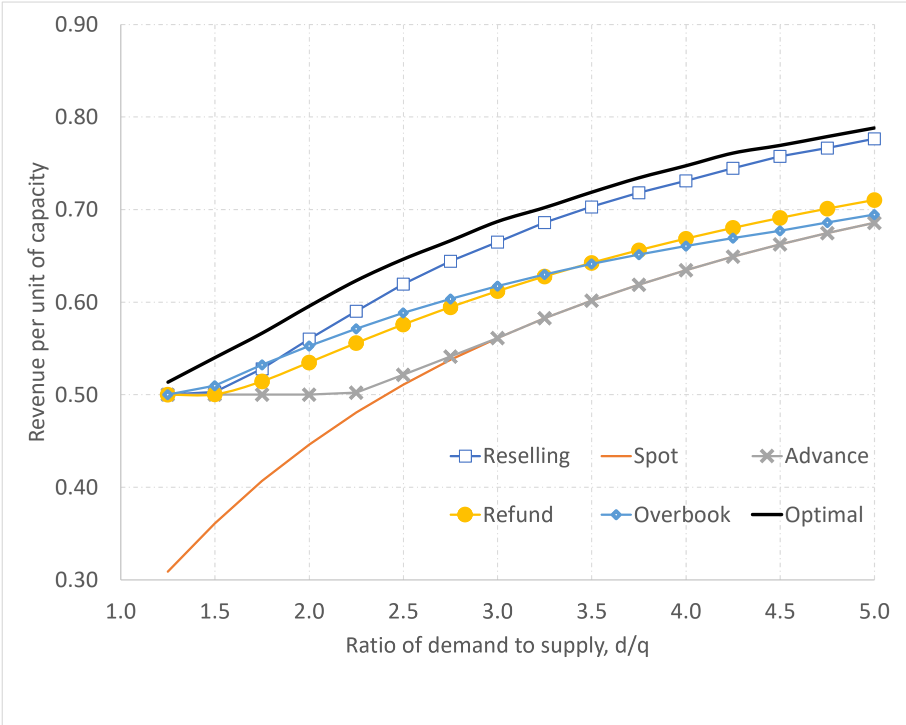

Figure 2. Small market performance, $q = 4$.

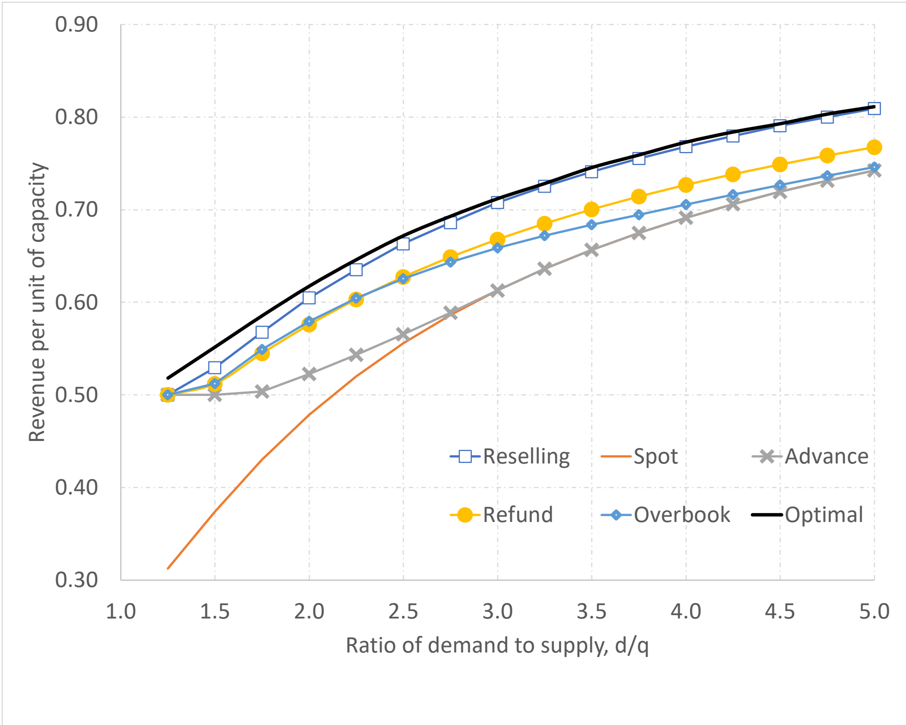

Figure 3. Small market performance with $q = 16$

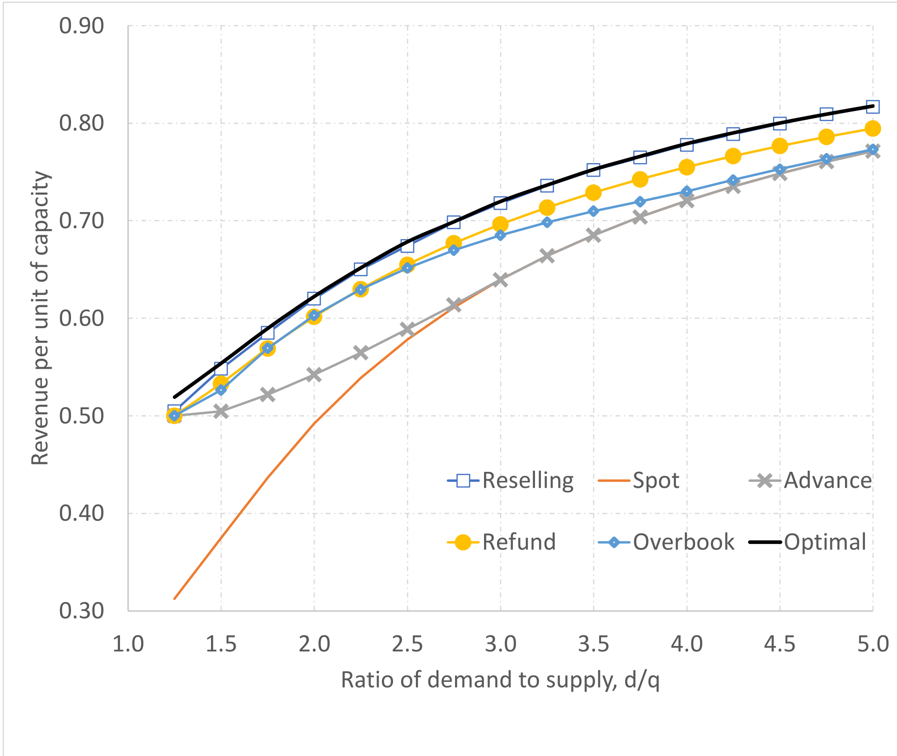

Figure 4. Small market performance with $q = 64$

across (almost) all d/q ratios and market scales, and it nearly replicates the optimal of the large market when $q = 64$. Only when the market is very small ($q = 4$) and demand is limited ($d \leq 7$) is reselling bested (by overbooking). In those situations the market is sufficiently thin that it can be hard to find a good clearing price in the reselling market.

Spot selling always improves its relative performance when demand increases because spot selling is effective at consumer selection (matching units to customers with the highest value), while its weakness is capacity utilization. Although demand-to-quantity ratios like $d/q > 2$ are desirable for maximizing revenue *given* the firm's capacity in the short term, unless building and maintaining capacity is very expensive, the firm is likely to benefit in the long run from capacity that more closely aligns with demand, i.e., $d/q < 2$. Given the sharp revenue decline with spot selling as d/q decreases, it may not be economical for the firm to expand capacity if it insists on using spot selling. However, with advance selling, the firm is better able to support capacity that approaches demand (i.e., low values of d/q).

Some form of recourse improves revenue considerably relative to advance selling without recourse no matter the size of the market. As expected, as the market becomes "larger", the recourse mechanisms perform similarly because there is limited aggregate uncertainty. Nevertheless, even with $q = 64$, reselling generates between 0.9 and 3.2% more revenue than the best of the other recourse mechanisms. It generates between 62% (when $d/q = 1.25$) and 6% (when $d/q = 5$) more revenue than spot selling.

6 Discussion

Our model is intentionally parsimonious to illustrate key distinguishing factors across selling mechanisms. Nevertheless, several extensions are worth consideration.

In our model a consumer's value does not depend on when they purchase the unit. In other words, consumers are patient. In practice, some consumers value knowing in advance that they have access to a service. For example, if a consumer is unable to purchase the unit in period 1 then the consumer may choose to not participate in the period 2 market, or may value period 2 participation less. In the context of our model, this preference would reduce the utility consumers receive from waiting to purchase, which allows the firm to increase its period 1 price. Hence, this preference for early resolution, beyond the issue of rationing risk, makes advance selling more attractive (with or without recourse) relative to spot selling.

Two other features of consumer preferences are important. First, the firm knows the distribution of the consumers' preferences precisely, which allows the firm to set a period 1 price equal to the amount all consumers are willing to pay. If there is actually some variation in the consumers' expected value (and they know it), then firm would need to be somewhat cautious with its period 1 price to avoid alienating too many period 1 consumers. Such uncertainty reduces the effectiveness of all of the advance selling options.

Second, it is presumed that the firm knows all consumers learn their value for the good between period 1 and 2. But what if such precise knowledge isn't possible? For example, say in period 1 there is a β probability that a consumer has already observed their value for the good. Our analysis assumes $\beta = 0$. But if buyers already know their value at the start of period 1 and yet the firm chooses a price p_1 that presumes the buyer is still uncertain, then the firm is choosing a price that is too low - the firm is offering a discount to compensate for preference uncertainty that may not exist. See Authors (2020) for a full analysis of that situation.

In our model with refunds and overbooking, the firm does not restrict the number of units that consumers can return. This creates the possibility that the firm accepts more units back than it would like. To avoid those situations, the firm could set a cap for the number of units it agrees to buy back. With refunds, the firm announces a cap along with a period 1 price and a buyback r at the start of period 1. With overbooking, the firm sets a cap along with the buyback r after observing overbook demand. The ability to limit the number of returns serves as an additional control and naturally (weakly) increases revenue. Nevertheless, the use of a cap also dampens a consumer's value from owning a unit because they can anticipate that they may not be able to return their unit even at the posted returned rate. In fact, we find that adding a cap to the basic overbooking or refund mechanism does not improve revenue significantly. Averaging over cases for which it is optimal to restrict returns, revenue increases by 0.26% with refunds (with a maximum increase of 1.04%) and by 1.8% (with a maximum increase of 3.24%) with overbooking. This suggests that it is nearly sufficient for the firm to merely choose the refund rate or the overbooking buy-back rate.

In the large market there are several mechanisms, including reselling, that can replicate the optimal profit. Performance diverges across the recourse mechanisms in the small market because they differ in how they utilize market information, with reselling as the top performer. However, if reselling is the best recourse mechanism, why don't firms always allow resale in practice? We offer several explanations.

Somewhat limited demand and supply: When demand is only somewhat larger than capacity and there are few units of capacity, advance selling provides a substantial boost in revenue over spot selling, but none of the recourse mechanisms yield significant improvements over basic advance selling.

Timing: In our model consumers observe their values sufficiently in advance of consumption to allow for a market to occur in which it is possible to transfer units. If consumers tend to observe their value sufficiently late (i.e., close to the time of consumption), there may not be enough time to effectively locate another buyer, which would reduce the value of selling in advance and surely the value of conducting a resale market. For example, in the airline industry consumers may observe their updated value only days or even hours before the flight, which may leave little time for an efficient resale market to work. Furthermore, if as suggested above, consumers have a preference for knowing in advance that they have access to the good, then there may be a limited number of consumers willing to participate in the period 2 resale market. In these situations, overbooking or refunds may be preferred. With either mechanism the firm retains more control over the transfer between consumers, enabling a faster exchange that is useful especially if consumers learn their preferences late.

Transaction costs: We have not included actual transaction costs, which are costs incurred by the system that strictly reduce total surplus. (A transfer between buyers and the firm does not reduce total surplus, but actual transaction costs do.) In practice, reselling requires consumer interaction and an ability for sellers to find potential buyers. To facilitate these matchings, firms can operate resale websites (this is commonly done by sport teams–all four major leagues now have sponsored resale marketplaces) or allow the use of third-party platforms (e.g., Stubhub and Ticketmaster), all of which involve various actual costs. Refunds require some communication between consumers and the firm. Overbooking does not involve matching costs, and the communication costs are minimal, but it does impose some costs on those willing to relinquish their unit. Any of these transaction costs introduce market inefficiencies that reduce the value of a recourse mechanism. In addition, the final ranking of these mechanisms could change depending on the differences in transaction costs. For example, if actual transaction costs with overbooking are substantially lower than with reselling, overbooking could be preferred. And if transaction costs across all recourse mechanisms are too high, the firm might prefer a mechanism without recourse.

Restrictions on transfer fees: Refunds and overbooking create indirect transfer fees by limiting the compensation for units returned back to the firm. Reselling uses direct transfer fees, which requires some control over the reselling market, either through ownership or contractual relations with a third-party that can prevent unauthorized transfers among consumers. Any limit on the firm's ability to design and regulate the reselling market therefore hinders the effectiveness of this mechanism. That said, any restriction on the feasible return rates with refunds or overbooking (e.g., a minimum level of compensation) would limit their effectiveness as well.

Consumer capability: Our consumers are able to participate optimally in a reselling market. In practice, sellers post prices and may lack the information or the needed skill to evaluate the best price. The resulting market may not be fully efficient (maximize the number of trades). Furthermore, consumers may not be able to fully anticipate the value of reselling. For example, the endowment effect (Kahneman *et al.* 1990) predicts that once ownership is acquired the buyer may value having it more. The buyer is unlikely to anticipate this endowment effect, thereby denying the firm access to some value.

One concern that is often raised regarding reselling markets is the possibility of competition between the firm and consumers. As illustrated in the large market, this should not be a concern when the firm has sufficient control over the design of the reselling market and is not constrained in its period 1 pricing. In period 2, each unit the firm possesses competes with the other units the firm possesses as well as the units held by consumers. The firm is willing to accept anything for its units because it has no value for them, whereas consumers are less price aggressive given that they have the option to retain the unit and earn some value. Consequently, each unit the firm sells in period 1 reduces the number of period 2 buyers and makes the average seller in period 2 less demanding (fewer units held by the firm that has no value and more held by consumers who have some value for the good), both of which help to raise the period 2 price. In sum, the firm is better off the more units are on the reselling market and not in the firm's possession.

7 Conclusion

We study how a firm should price its limited perishable capacity over time. We find (in a large market) that it can be optimal for a firm to allow consumers to buy in advance and to resell to other consumers, as in ticket exchanges that are now common in many markets. Although the negative impact of speculators is often feared, we find that in an efficient reselling market speculators do not participate and should be of no concern.

Other recourse mechanisms, such as refunds/options or overbooking, can improve upon spot selling or advance selling without recourse, but are inferior to reselling unless the market is very large. They are not as effective because they do not exploit market information as well as a reselling market. Equivalently, they each force the firm to make decisions with only partial information. Although refunds provide some downside insurance to consumers, the firm still must choose a spot period price without fully knowing demand. Overbooking reveals to the firm the number of high-valued customers, but leaves the firm to guess the number willing to relinquish their good. An efficient reselling market adjusts prices to account for the number of consumers at either end of the market.

Clearly, recourse strategies can only be effective if actual transaction costs to implement them are not excessive. In markets with high transaction costs, recourse strategies might not be desirable over simpler advance selling without recourse. However, the use of information technology generally reduces transaction costs, making these recourse strategies potentially feasible and even highly profitable. This is consistent with the observation that instead of trying to prevent resale, many sellers of perishable capacity (e.g., sports teams) now actively encourage reselling among their consumers.

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A Large Market Analysis

A.1 Spot and Advance selling

Spot selling occurs when the firm does advance selling without any advanced sales, $q_1 = 0$. Hence, the two mechanisms can be analyzed in the same framework.

A consumer who purchases in period 1 earns an expected utility of $1/2 - p_1$. Alternatively, the consumer could wait to attempt to purchase in period 2 at price p_2. The expected value of waiting given a period 2 price of p_2 is $(1-p_2)^2/2$: there is a $1-p_2$ probability the consumer has a utility high enough to be willing to purchase at p_2 and conditional on that, the expected gain in utility over the price paid is $(1-p_2)/2$. Hence, when consumers expect p_2 to be the period 2 price, the most the firm can charge in period 1 is

$$p_1 \leq \frac{1}{2} - \frac{(1-p_2)^2}{2} = \frac{1}{2}p_2(2-p_2).$$

In period 2 available demand is $d - q_1$ and available supply is $q - q_1$. The quantity sold is $q_2 = \min\{(1-p_2)(d-q_1), q-q_1\}$ and revenue is

$$r_2 = p_2 q_2 = \begin{cases} p_2(q-q_1) & p_2 < \frac{d-q}{d-q_1} \\ p_2(1-p_2)(d-q_1) & \frac{d-q}{d-q_1} \leq p_2 \end{cases}$$

Total revenue is

$$r = p_1 q_1 + r_2 = \begin{cases} p_2\left[q - \frac{1}{2}p_2 q_1\right] & p_2 < \frac{d-q}{d-q_1} \\ p_2\left[(1-p_2)d + \frac{1}{2}p_2 q_1\right] & \frac{d-q}{d-q_1} \leq p_2 \end{cases}$$

When the firm's period 2 supply is constraining (the first case), the firm's revenue is strictly decreasing in q_1. It follows that $q_1 = 0$ is optimal and it is sufficient to consider the second case. When the firm's period 2 supply is not constraining (the second case), the firm's revenue is strictly increasing in q_1, which implies

$$q_1 = \left(d - \frac{d-q}{p_2}\right)^+$$

The firm's revenue is maximized in the feasible range with

$$p_2 = \max\left\{\frac{d+q}{2d}, \frac{d-q}{d}\right\}$$

The optimal revenue, period 2 price and period 1 quantity are ($\phi = q/d$)

$$r^* = \begin{cases} (1-\phi)\,q & \phi < \frac{1}{3} \\ \frac{1}{8}\frac{(1+\phi)^2}{\phi}q & \phi \geq \frac{1}{3} \end{cases}$$

$$p_2^* = \begin{cases} 1-\phi & \phi < \frac{1}{3} \\ \frac{1+\phi}{2} & \phi \geq \frac{1}{3} \end{cases}$$

$$q_1^* = \begin{cases} 0 & \phi < \frac{1}{3} \\ \frac{(3\phi-1)}{\phi(1+\phi)}q & \phi \geq \frac{1}{3}. \end{cases}$$

When $\phi < 1/3$ the firm sells nothing in advance, effectively operating with spot selling. When $\phi > 1/3$, demand is limited and selling at the revenue maximizing price leaves some units unsold.

A.2 Optimal Mechanism

Proof of Theorem 1. In period 1 there is only one type of buyer and for each consumer either a unit is transferred to the consumer or not. In period 2 there are two types of consumers, ones that purchased in period 1, which are referred to as the "sellers" and those that did not, which are referred to as the "buyers".

The period 1 decision is simple. Either a unit is transferred to a consumer for a fee or not. Let q_1, $q_1 \leq q$, be the number of units transferred. As all consumers are identical in this period (i.e., there is no private information), it is sufficient to consider a single transfer price.

In period 2 sellers either keep their unit or return it to the firm for some payment. Buyers can receive a unit for some fee. Consumers are distinguished only by their reported value and whether they possess a unit or not. Thus, in a truth inducing mechanism, there can only be a single price offered to buyers, p_2: if there were two or more prices assigned to the reported values from the buyers then they all would have an incentive to report a value associated with the lowest price. Similarly, there can only be a single price offered to the sellers. However, those two prices need not be identical. Furthermore, it is sufficient to consider only one sell price and one buy price because the market outcomes are fully predictable. (For many optimal mechanisms, like an auction, the sell and buy prices could be contingent on market outcomes if no agent could gain by misreporting. In this case there is only one contingency, so only one pair of prices is possible.) Hence, without loss of generality, let $p_2 - t$ be what each seller receive for units they return to the firm, where t is referred to as the transfer fee.

Start with the second period. There are $d - q_1$ buyers without a unit and q_1 sellers with a unit. Buyers who report value p_2 or higher receive a unit and pay p_2. Sellers who report a value $p_2 - t$ or lower relinquish their unit and receive $p_2 - t$. The firm keeps t per unit transferred between a buyer and a seller.

The firm chooses the parameters of the mechanism (e.g., p_2, t, and transfer quantities) to maximize revenue in this period subject to capacity constraints (units cannot be sold that cannot be produced). The firm could eliminate any transfers in period 2 by setting a sufficiently high transfer fee, i.e., $p_2 \leq t$. This is equivalent to operating with advance selling without recourse. Alternatively, the firm chooses a t such that there may be some transfers from sellers to buyers. It is sufficient to continue assuming $t \leq p_2$ (because $p_2 = t$ implements advance selling without recourse).

Given that some transfers are feasible (i.e., $t \leq p_2$), with the optimal mechanism supply matches demand in period 2. To explain, if supply were less than demand, then an increase in p_2 or t raises the firm's revenue while still satisfying all demand. If supply were more than demand then the firm could increase t, thereby reducing supply and increase its revenue. Thus, supply must match demand. It follows that the firm sells its remaining supply in period 2.

Let $q_2 = q - q_1$ be the firm's supply in period 2. Given that supply in period 2 matches demand,

$$(d - q_1)(1 - p_2) = q_2 + q_1 (p_2 - t)$$

which implies the period 2 price is

$$p_2 = \frac{d - q + q_1 t}{d}$$

The firm's revenue in period 2 is $r_2 = p_2 q_2 + t q_1 (p_2 - t)$.

It remains to determine the period 1 price p_1, the transfer fee t, and the number of units sold q_1 in period 1. If a consumer does not purchase in period 1 then the consumer earns expected value $(1 - p_2)^2/2$ in period

2. Alternatively, the consumer can request to purchase in period 1 and receive expected value

$$(p_2 - t)^2 + (1 - (p_2 - t))(1 + (p_2 - t))/2$$

The first term is expected value when the consumer sells and the second term is the expected value when the consumers holds on to the unit. Because in period 2 supply exactly equals demand, all period 1 buyers with value less than $p_2 - t$ are assured to get at least value $p_2 - t$.

The period 1 price that makes the consumers indifferent between requesting to purchase in period 1 or waiting is

$$p_1 = (1 - t)\, p_2 + \frac{1}{2}t^2$$

The period 1 price is decreasing in t (recall, $t \leq p_2$): imposing a larger transfer fee naturally reduces the amount consumers are willing to pay in advance. If there is no transfer fee, $t = 0$, then consumers in period 1 are willing to pay the period 2 price, $p_1 = p_2$, because when there is no uncertainty in prices and supply always matches demand (i.e., there is no rationing risk), then purchasing in advance with a utility floor of p_2 is equivalent to waiting to period 2 with the option to purchase at p_2.

The firm's total revenue is

$$R = p_1 q_1 + r_2 = q\left(\frac{d-q}{d}\right) + q_1\left(\frac{q}{d}t - \frac{1}{2}t^2\right).$$

Assuming $q_1 > 0$, the optimal transfer fee is $t^* = \phi$. Given that fee, it is optimal for the firm to sell its entire supply in period 1, $q_1 = q$. Overall, $q_1 = q$ and $t = t^*$ is optimal. □

B Small Market Analysis

Let $b(x, n, s)$ and $B(x, n, s)$ be the binomial density and distribution functions for x successes from n attempts with a success probability s.

Let $S(k, n, p)$ be the firm's expected sales when k units are for sale to $n > k$ consumers at price p. That is, $S(k, n, p)$ is the minimum of k and demand at price p,

$$\begin{aligned} S(k, n, p) &= \sum_{i=0}^{k} i \binom{n}{i} (1-p)^i p^{n-i} + k\left(1 - \sum_{i=0}^{k} \binom{n}{i} (1-p)^i p^{n-i}\right) \\ &= npB(k-1, n-1, 1-p) + k(1 - B(k, n, 1-p)), \end{aligned}$$

where the last equality follows from the fact that for the binomial distribution

$$\sum_{x=x_l}^{x_h} xb(x,n,s) = ns\left(B(x_h-1,n-1,s) - B(x_l-2,n-1,s)\right).$$

B.1 Optimal Mechanism

Proof of Theorem 2. The proof modifies the proof of the dynamic optimal mechanism in Ely *et al.* (2017), which assumes that customers are heterogeneous in period 1. Let $q_1 \in \{0, ..., q\}$ be the number of units purchased in period 1. Because all customers are homogeneous in period 1, the firm can choose a period price $p_1(q_1)$ that makes them indifferent between purchasing the unit or waiting and therefore ensure that all q_1 units that are offered for sale get sold. For convenience of exposition, let subscripts j denote customers that hold units in period 2 ("sellers") and let subscripts k denote customers who do not ("buyers").

In period 2, each customer observes their realized value. A direct revelation mechanism is described by a collection of functions $(\phi, \mathbf{t})$, such that, for each q_1, $\phi^{q_1}(\mathbf{v}) = \left(\phi_i^{q_1}(\mathbf{v})\right)_{i=1}^n$ is the probability that each customer i has the unit and $\mathbf{t}^{q_1}(\mathbf{v}) = \left(t_i^{q_1}(\mathbf{v})\right)_{i=1}^n$ is the payment that each customer i makes to the firm in period 2. A negative payment is possible and implies a payment that the firm makes to a customer.

Consider a customer i who truthfully reports his value v_i, while other customers report $\mathbf{v}_{-i}$, and suppose the number of units sold in period 1 is q_1. Disregarding the period 1 price, p_1, the payoff to customer i is $U_i^{q_1}(\mathbf{v}) = \phi_i^{q_1}(\mathbf{v})v_i - t_i^{q_1}(\mathbf{v})$. The mechanism is incentive compatible if, for all q_1, i, v_i, $\hat{v}_i$ and $\mathbf{v}_{-i}$,

$$U_i^{q_1}(v_i, \mathbf{v}_{-i}) \geq \phi_i^{q_1}(\hat{v}_i, \mathbf{v}_{-i})v_i - t_i^{q_1}(\hat{v}_i, \mathbf{v}_{-i}).$$

Denote the expected utility of a customer who has a unit (gross of the unit price p_1), given that q_1 units were sold by $\pi_a(q_1)$ and of a customer who does not hold a unit by $\pi_b(q_1)$. These expected payoffs are :

$$\pi_a(q_1) = \mathbb{E}\left[U_j^{q_1}(\tilde{\mathbf{v}})\right]; \quad \pi_b(q_1) = \mathbb{E}\left[U_k^{q_1}(\tilde{\mathbf{v}})\right].$$

The expected per-customer profit earned in period 2 from customers who hold and don't hold units given that q_1 tickets were sold is given by

$$\bar{\pi}(q_1) = \mathbb{E}\left[t_j^{q_1}(\tilde{\mathbf{v}})\right] \quad \underline{\pi}(q_1) = \mathbb{E}\left[t_k^{q_1}(\tilde{\mathbf{v}})\right].$$

In period 1, the expected payoff to a customer i who purchases a unit at price p_1 is $\pi_a(q_1) - p_1$. The expected payoff to this customer if waiting is $\pi_b(q_1)$. In an optimal pricing mechanism, the unit price p_1 must satisfy:

$p_1(q_1) = \pi_a(q_1) - \pi_b(q_1)$.

The firm's expected revenue is therefore

$$\begin{aligned} R(q_1) &= p_1(q_1)\, q_1 + q_1 \bar{\pi}(q_1) + (d - q_1)\, \underline{\pi}(q_1) \\ &= q_1 (\pi_a(q_1) - \pi_b(q_1)) + q_1 \bar{\pi}(q_1) + (d - q_1)\, \underline{\pi}(q_1) \\ &= q_1 (\pi_a(q_1) + \bar{\pi}(q_1)) + (d - q_1) \left(\underline{\pi}(q_1) - \frac{q_1}{d - q_1} \pi_b(q_1) \right). \end{aligned}$$

The period 2 mechanism must satisfy, for any q_1, any customer i, and any $\mathbf{v}$,

$$U_i^{q_1}(\mathbf{v}) = U_i^{q_1}(1, \mathbf{v}_{-i}) - \int_{v_i}^{1} \phi_i^{q_1}(y, \mathbf{v}_{-i})\, dy.$$

The share of the surplus a customer i expects to obtain in period 2, for each q_1 and $\mathbf{v}_{-i}$, is determined up to a constant by their value v_i and the allocation rule ϕ. For sellers, adjusting this constant does not affect expected payoffs in period 1 provided the period 1 price, p_1, is correspondingly adjusted. It is therefore sufficient to focus on period 2 mechanisms such that for all q_1, all sellers j, and all $\mathbf{v}_{-j}$, $U_j^{q_1}(1, \mathbf{v}_{-j}) = 1$. Adjusting period 2 payoffs for buyers, however, does affect ex-ante payoffs. If the payoff earned by the customer with the minimum value $U_k^{q_1}(0, \mathbf{v}_{-k})$ is positive, the firm benefits from increasing buyer payments and can do so by choosing the appropriate constant. Therefore, it is sufficient to focus on mechanisms that satisfy $U_k^{q_1}(0, \mathbf{v}_{-k}) = 0$. The corresponding transfer prices are given by

$$\begin{aligned} t_j^{q_1}(v_j, \mathbf{v}_{-j}) &= v_j \phi_j^{q_1}(v_j, \mathbf{v}_{-j}) + \int_{v_j}^{1} \phi_j^{q_1}(y, \mathbf{v}_{-j})\, dy - 1, \\ t_k^{q_1}(v_k, \mathbf{v}_{-k}) &= v_k \phi_k^{q_1}(v_k, \mathbf{v}_{-k}) + \int_{0}^{v_k} \phi_k^{q_1}(y, \mathbf{v}_{-k})\, dy \end{aligned}$$

implying that revenue can be expressed through the allocation rule ϕ alone. It remains to derive the optimal allocation.

The revenues earned from a seller j and a buyer k are

$$\begin{aligned} \bar{\pi}(q_1) &= \int_{\mathbf{v}} t_j^{q_1}(\mathbf{v})\, dF(\mathbf{v}) = \int_{\mathbf{v}} \left[v_j \phi_j^{q_1}(\mathbf{v}) - U_j^{q_1}(\mathbf{v}) \right] dF(\mathbf{v}) \\ &= -\int_{\mathbf{v}_{-j}} U_j^{q_1}(1, \mathbf{v}_{-j})\, dF(\mathbf{v}_{-j}) + \int_{\mathbf{v}} \phi_j^{q_1}(\mathbf{v})\, v_j\, dF(\mathbf{v}) + \int_{\mathbf{v}_{-j}} \left[\int_{v_j} F(v_j)\, \phi_j^{q_1}(\mathbf{v})\, dv_j \right] dF(\mathbf{v}_{-j}) \end{aligned}$$

and

$$\underline{\pi}(q_1) = \int_{\mathbf{v}} t_k^{q_1}(\mathbf{v})\, dF(\mathbf{v}) = \int_{\mathbf{v}} \left[v_k \phi_k^{q_1}(\mathbf{v}) - U_k^{q_1}(\mathbf{v}) \right] dF(\mathbf{v})$$

and the utilities of customers with and without units are

$$\begin{aligned}\pi_a(q_1) &= \int_{v_j}\int_{\mathbf{v}_{-j}} U_j^{q_1}(\mathbf{v})\,dF(\mathbf{v}_{-j})\,dF(v_j)\\ &= \int_{\mathbf{v}_{-j}} U_j^{q_1}(1,\mathbf{v}_{-j})\,dF(\mathbf{v}_{-j}) - \int_{\mathbf{v}_{-j}}\left[\int_{v_j} F(v_j)\,\phi_j^{q_1}(\mathbf{v})\,dv_j\right]dF(\mathbf{v}_{-j})\end{aligned}$$

and

$$\pi_b(q_1) = \int_{v_k}\int_{\mathbf{v}_{-k}} U_k^{q_1}(\mathbf{v})\,dF(\mathbf{v}_{-k})\,dF(v_k)\,.$$

Adding the expressions, we obtain

$$q_1\left(\bar{\pi}(q_1) + \pi_a(q_1)\right) = \mathbb{E}_{\tilde{\mathbf{v}}}\left[\sum_{j=1}^{q_1}\phi_j^{q_1}(\tilde{\mathbf{v}})\,\overline{VS}(\tilde{v}_j)\right], \tag{1}$$

where $\overline{VS}$ is the virtual surplus of a seller, $\overline{VS}(v_j) = v_j$. Using the fact that buyers with the lowest values earn zero surplus, we have

$$(d-q_1)\left(\underline{\pi}(q_1) - \frac{q_1}{d-q_1}\pi_b(q_1)\right) = \mathbb{E}_{\tilde{\mathbf{v}}}\left[\sum_{k=q_1+1}^{d}\phi_k^{q_1}(\tilde{\mathbf{v}})\,\underline{VS}(\tilde{v}_k)\right], \tag{2}$$

where $\underline{VS}$ is the virtual surplus of a buyer,

$$\underline{VS}(v_k) = 2v_k - 1 - \frac{q_1}{d-q_1}(1-v_k) = \left(2 + \frac{q_1}{d-q_1}\right)v_k - \frac{d}{d-q_1}.$$

Combining (1) and (2), we obtain an expression for the revenue in an optimal mechanism implementing an allocation rule ϕ as a function of q_1:

$$R(q_1) = \mathbb{E}_{\tilde{\mathbf{v}}}\left[\sum_{j=1}^{q_1}\phi_j^{q_1}(\tilde{\mathbf{v}})\,\overline{VS}(\tilde{v}_j) + \sum_{k=q_1+1}^{n}\phi_k^{q_1}(\tilde{\mathbf{v}})\,\underline{VS}(\tilde{v}_k)\right].$$

The transformed expected revenue is the familiar expected virtual surplus.

It is possible to implement the optimal mechanism with a double auction. Specifically, for any seller's value v_j, define the matching value $v_k(v_j)$ for a buyer as $\overline{VS}(v_j) = \underline{VS}(v_k(v_j))$:

$$v_k(v_j) = \frac{d-q_1}{2(d-q_1)+q_1}v_j + \frac{d}{2(d-q_1)+q_1}.$$

Similarly, for every buyer value v_k, define $v_j(v_k)$ as the matching value of the seller:

$$v_j(v_k) = \frac{2(d-q_1)+q_1}{d-q_1} v_k - \frac{d}{d-q_1}.$$

The rules of the double auction are as follows: Each customer submits a bid. Sellers with value v_j bid v_j. Buyers with value v_k, bid $v_j(v_k)$. (Following arguments in Ely *et al.* (2017), it is possible to show that this bidding is a dominant strategy equilibrium.) The d customers are ranked in descending order of bids. Define y as the minimum between the number of units, q, and the number of positive bids. The y highest bidding customers are allocated a unit and the remaining $d-y$ customers are not. This implies that there are four groups of consumers: "sellers" who are in the top y bids keep their unit, "buyers" in the top y bids get a unit (they buy), "sellers" in the bottom $d-y$ bids give up their unit (they sell) and "buyers" in the bottom $d-y$ bids remain without a unit. Observe that if $q_1 = q$, an optimal allocation rule always allocates all q units, because $\overline{VS}(v_j) \geq 0 \;\; \forall v_j$, but if $q_1 < q$, it is possible that $y < q$, so that the firm does not allocate all units. Payments are determined as follows: Let b^y and b^{y+1} denote the y and $(y+1)^{st}$ highest bids. Any customer who sells a unit receives compensation equal to b^y (which is positive by the definition of y). Any customer who buys a unit is charged $v_k\left(b^{y+1}\right)$, which equals the $y+1^{st}$ customer's value, if that customer is a buyer and is greater than the $y+1^{st}$ customer's value, if that customer is a seller. Observe that there is no clear ranking between b^y and $v_k\left(b^{y+1}\right)$ (i.e., both $b^y > v_k\left(b^{y+1}\right)$ and $b^y < v_k\left(b^{y+1}\right)$ may hold). The transfers are zero for all other customers (those who bought a unit and keep it and those who did not buy a unit and remained without one). □

B.2 Spot Selling

Differentiating $R_s(q,d,p)$ with respect to p, we have $R_s'(p;q,d) = S(p) + pS'(p)$. Let $\tau(p) = R'(p)$. Using the fact that

$$\frac{d}{dp} B(k,n,p) = -n \binom{n}{k} p^k (1-p)^{n-k}$$

and differentiating the sales function, $S(p)$, we have

$$\begin{aligned} S'(p) &= nB(k-1,n-1,1-p) - np(n-1)\binom{n-1}{k-1} p^{k-1}(1-p)^{n-k} + kn\binom{n}{k} p^k (1-p)^{n-k} \\ &= nB(k-1,n-1,1-p) + n\binom{n-1}{k-1} p^k (1-p)^{n-k}. \end{aligned}$$

Because $R_s(0) = R_s(1) = 0$, $\lim_{p\to 0} \tau(p) = S(0) > 0$ and $\lim_{p\to 1} \tau(p) = S'(1) = 0$, it remains to show that there is at most one $p \in (0,1)$ that solves $\tau(p) = 0$. Rearranging $\tau(p) = 0$, we have:

$$\frac{S(p)}{p} = -S'(p)$$

$S(p)/p$ decreases in p. Therefore, it is sufficient to show that $S'(p)$ decreases in p. Let $y = np^{k-1}(1-p)^{n-k}$.

$$\begin{aligned} S''(p) &= -(n-1)\binom{n-1}{k-1} y + \binom{n-1}{k-1}\left(k - (n-k)\frac{p}{1-p}\right) y \\ &= \binom{n-1}{k-1} y \left(-(n-k)\frac{p}{1-p} - (n-1) + k\right) < 0, \end{aligned}$$

because $k \leq n-1$. Therefore, $R_s(q,d,p)$ is unimodal and there exists a unique p_s that maximizes spot revenue.

B.3 Refunds/Options

Given a buyback price r, between 0 and q_1 units are returned from the period 1 customers, leaving the firm with between $q - q_1$ and q units to price in period 2. The expected value a buyer earns if they choose to wait in period 1 (i.e., not purchase) is

$$\mathbb{E}\left[\omega\left(p_s^*\left(q_2(r), d - q_1\right)\right)\right] = \sum_{q_2 = q - q_1}^{q} b\left(q_2 - q + q_1, q_1, r\right) \omega\left(p_s^*\left(q_2, d - q_1\right)\right).$$

B.4 Overbooking

Let $\pi_o(q_1, p_2, d_o, r)$ be the firm's profit from overbook sales,

$$\pi_o(q_1, p_2, d_o, r) = \begin{cases} p_2 S(d_o, q_1, 1-r) - q_1 r^2 & d_o < q_1 \\ q_1 r (p_2 - r) & d_o \geq q_1 \end{cases}$$

Let $\pi_o^*(q_1, p_2, d_o) = \max_r \pi_o(q_1, p_2, d_o, r)$. The profit from overbook sales is analogous to the spot selling one with stochastic supply instead of stochastic demand. Hence, there exists a unique $r_o^*(q_1, p_2, d_o)$ that maximizes π_o.

Given q_1, the firm's period 2 revenue is

$$\begin{aligned} R_2(q_1,p_2) &= p_2 S(q-q_1,d-q_1,p_2) \\ &\quad + \sum_{d_o=1}^{q_1-1} b(d_o+q-q_1,d-q_1,1-p_2)\,\pi_o^*(q_1,p_2,d_o) \\ &\quad + (1-B(q-1,d-q_1,1-p_2))\,\pi_o^*(q_1,p_2,q_1)\,. \end{aligned}$$

A search determines the optimal $p_2^*(q_1)$ for each q_1.

Let $r_o^*(q_1,d_o) = r_o^*(q_1,p_2^*(q_1),d_o)$ and $R_2^*(q_1) = R_2(q_1,p_2^*(q_1))$. The expected earnings from waiting is

$$\pi^b(q_1) = \begin{cases} \omega(p_2^*(q))\,\mathbb{E}_{d_o}\left[(1-(1-r_o^*(q,d_o))^q)\right] & q_1 = q \\ \omega(p_2^*(q_1)) & q_1 < q, \end{cases}$$

where conditional on the buyer obtaining value from the unit, the probability that $d_o = \delta$ (and therefore the buyback price is $r_o^*(q,\delta)$) is $b(\delta-1,d-q-1,1-p_2)$.

The expected earnings from buying in period 1 is

$$\begin{aligned} \pi^a(q_1) &= \frac{1}{2}\mathbb{E}_{d_o}\left[(r_o^*(q_1,d_o))^2\right] \\ &= \sum_{d_o=1}^{d-q} b(d_o+q-q_1,d-q_1,1-p_2^*(q_1))\,\frac{(r_o^*(q_1,d_o))^2}{2}. \end{aligned}$$

B.5 Reselling

Evaluation of the clearing prices, trade quantities, and revenues under all conditions is feasible, but complex. We choose to evaluate the reselling mechanism via simulation. As $q_1 = q$ is optimal in the large market and in the optimal mechanism (based on our numerical results), we consider only reselling mechanisms with $q_1 = q$. We search for the best transfer precentage, t, on an evenly spaced 100-point grid on the $[0,1]$ interval.