Green Incentives in Decentralized Consortia

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Abstract

Problem Definition: Environmentally conscious consumers create incentives for firms to market sustainable products, but the environmental quality of such products is difficult for consumers to verify. Decentralized consortia offer a new certification approach in which member firms validate one another's sustainability claims and record outcomes on distributed ledgers. We study whether such consortia strengthen firms' sustainability incentives and how they affect profits, consumer surplus, and social welfare. **Methodology/results:** We develop a game-theoretic model in which firms choose costly sustainability effort that increases the probability their products are genuinely green, while consumers are heterogeneous in their willingness to pay for green products. We compare a benchmark without peer verification to a consortium setting in which another member validates the product with imperfect accuracy. Joining the consortium always improves firm profit by enabling state-contingent pricing based on certification outcomes. However, certification also exposes firms to downside risk: if sustainability effort does not result in a certifiable green product, the firm must sell at a discount. When baseline sustainability effort is already high, certification has limited room to further increase the certified-state premium, while exposing the firm to downside risk if the product fails certification. In such markets, which often correspond to high consumer willingness to pay for green attributes, consortium adoption can reduce equilibrium sustainability effort. The qualitative message persists when firms have intrinsic sustainability preferences and when consortium members compete in the product market. **Managerial implications:** Decentralized consortia are not uniformly welfare-enhancing. They are most likely to improve sustainability outcomes when baseline effort is low, often in markets with weaker green demand. In high-WTP markets where baseline effort is already high, they can raise firm profits while weakening sustainability incentives. Managers and policymakers should evaluate market conditions, certification accuracy, and competition before promoting or regulating peer-verification consortia.

Keywords: Certification, Distributed Ledgers, Eco-conscious Consumers, Environmental Responsibility, Sustainability.

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1 Introduction

In the race to attract environmentally conscious consumers, firms face a critical tension: while green product claims can command premium prices, verifying the authenticity of these claims remains a challenge. Decentralized consortia offer a novel approach to validate these claims through peer-verification networks, but it is unclear whether these networks genuinely strengthen sustainability incentives or instead create a new façade of legitimacy. This paper examines this question in an operations management context, analyzing how consortium networks affect profits, welfare, and sustainability efforts.

The emergence of environmentally conscious consumers has become a significant driving force for firms racing to adapt to sustainability standards and produce 'green' products (CDP 2019). Reports indicate that a significant portion of consumers base their purchase decisions on perceived sustainability, with many willing to pay a premium for such products (McKinsey & Company 2020, Simon-Kucher & Partners 2021). In the United States, the market for environmentally conscious products is valued at over $150 billion (Nielsen 2021), highlighting the potential impact of sustainability on demand and profitability. However, the environmental quality of products is not readily observable to consumers, making it difficult to distinguish genuinely sustainable products from those that are not (Delmas and Burbano 2011, Marquis et al. 2016). This has increased consumer skepticism and demand for credible verification and transparency from brands (GreenPrint 2022, Nygaard and Silkoset 2022).

To address this challenge, some brands are leveraging distributed ledgers (DLs) (e.g., public or private permissioned blockchain systems) to certify sustainability claims in a tamper-proof way. For example, luxury brands such as LVMH, Cartier, Prada, and Mercedes-Benz have founded the Aura Consortium to share information about their products' origin and environmental impacts, with the stated mission to "promote socially responsible, sustainable, and customer-centric business practices throughout the lifecycle of luxury products" (New York Times 2021, Aura Blockchain Consortium 2021).[1] Membership is thought to improve credibility by enabling transparent and immutable logging of detailed product information. Such information is securely verified by network participants and protected from alteration after being recorded (Sumkin et al. 2021, Gaur and Gaiha 2020, Pun et al. 2021). Despite their advantages, DLs are not without limitations. A key challenge is data input integrity, which refers to the possibility that while the data entered in the system is secure, it could be incorrect (Deloitte 2018, Chod et al. 2020). For instance, Hema Fresh, a Chinese

[1] See https://auraconsortium.com/.

grocery chain that uses a DL to track its food products, encountered discrepancies between the DL records and the actual products (Forbes 2019). Thus, it remains unclear how DL-enabled certifications impact the sustainability of products.

Given the advantages and potential weaknesses of DLs, we examine the impact of adopting decentralized consortia for certifying green products. Specifically, we aim to address the following questions:

1. How do decentralized consortia for certifying green products impact firm sustainability efforts?

2. What are the implications for consumer surplus and social welfare?

3. How does competition between consortium members affect these outcomes?

To address these questions, we construct a game-theoretic model of firms selling to strategic consumers who are heterogeneous in their valuation for green products. The firms can exert costly sustainability effort to increase the likelihood that their products end up being truly green. We analyze two separate games: in the "off-consortium" game, firms sell products directly to consumers without any peer verification; in the "on-consortium" game, firms form a consortium where each firm's product is verified by another member acting as the validator, who certifies the green status of the product and records the information on an immutable distributed ledger. The consortium verification is not perfect: a validator may have limited accuracy in verifying other firms' products, which may result from insufficient information, time, or effort. Consequently, there is a possibility that the consortium may certify products that do not perfectly match their actual green status.

Our results indicate that while the consortium structure increases firm profits, the effect of decentralized certification on firm's sustainability effort compared to a benchmark without certification is more nuanced. Certification enhances product trustworthiness because it increases consumers' expectations that the product is genuinely green. This allows the firm to charge a price premium for certified products, incentivizing higher sustainability effort to achieve certification and capture the premium. However, this positive effect does not tell the whole story. Certification also exposes the firm to downside risk: if the product is found to be non-green and fails certification, it must be sold at a discount, and the sustainability effort invested in the product is effectively "wasted". When a firm is already exerting high sustainability effort, the certified price premium has limited room to grow further and cannot offset this risk, a situation that typically arises in markets with high consumer willingness to pay for green products. In this case, firms have strong incentives to reduce their sustainability effort on the consortium. For consumers, this reduction in the firm's sustainability efforts can undermine the actual value delivered by the decentralized certification, resulting in reduced consumer surplus and creating an incentive conflict. While consortium membership improves firms' profits, the reduction in consumer surplus can outweigh the profit gains, resulting in a net loss of social welfare.

This high-level finding remains robust when firms incorporate intrinsic sustainability preferences into their objective and when firms compete in the product market. The latter case is particularly relevant, as members of consortia can and often do operate in overlapping segments. Under Hotelling competition between consortium members, the qualitative pattern carries over: the consortium uniformly raises firm profit, but may still reduce both sustainability effort and consumer surplus relative to the off-consortium benchmark. These results sharpen the cautionary message for consortium executives and regulators: even in competitive product markets, the profits firms gain from membership can come at the expense of sustainability and consumer welfare.

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 outlines the off-consortium game. Section 4 outlines the on-consortium game. In Section 5, we compare the outcomes between the two games. Section 6 presents robustness checks and extensions under relaxed modeling assumptions, and Section 7 concludes. All proofs are in the Appendix.

2 Literature Review

Our work is related to several streams of research: (i) impact of firm's environmental and social responsibility on its demand, (ii) certification and ecolabels, and (iii) operational impacts of blockchains and distributed ledgers. We review each stream separately.

First, consider the literature involving the impact of firm's environmental and social responsibility on its demand. Huang and Rust (2011) argue that consumers maximize their happiness by considering standard of living, psychological benefits from environmentally responsible behavior, and charity. There is empirical evidence that consumers are willing to pay for environmentally friendly products in general (Diederich and Goeschl 2014, Lanz et al. 2018, Golob and Kronegger 2019). Arora and Henderson (2007) indicate that the embedded price premium strategies for socially responsible products benefit firms by attracting purchases from eco-conscious consumers. Gao and Souza (2022) indicate that a low carbon offsetting price can effectively promote products with lower carbon footprint when eco-conscious consumers have higher valuations and willingness-to-pay for environmentally friendly products with social causes. Consistent with these insights, our model assumes that a firm's sustainability effort impacts demand from consumers, who are heterogeneous in their environmental consciousness levels. There is also some experimental evidence that a firm's transparent social responsibility positively impacts its demand (Pigors and Rockenbach 2016, Kraft et al. 2018, Buell and Kalkanci 2021). Several papers model the negative impact of unsustainable practices in the supply chain on demand, including unsustainable natural resource harvesting (Orsdemir et al. 2019), supplier-level factory safety violations (Plambeck and Taylor 2016), or supplier-level responsibility violations in general (Guo et al. 2016). Kalkanci and Plambeck (2020a) analyze the cost incurred by a firm to learn about a supplier's environmental practices, and how such costs impact the market valuation of a firm under different disclosure policies. Closely related to this stream is research that focuses on how firms can improve environmental social responsibility by influencing a supplier to behave responsibly. Such practices include a shared savings contract (Corbett and DeCroix 2001), extended producer responsibility (Huang et al. 2019), publishing a responsible suppliers list (Kalkanci and Plambeck 2020b), implementing responsible sourcing policies (Agrawal and Lee 2019), offering a wholesale price premium (Karaer et al. 2017), and jointly auditing suppliers with competitors (Chen et al. 2020). While prior work has focused on modeling the direct interactions between the firm and its suppliers, we instead focus on the firm's own decision to engage in sustainable production and, more specifically, on how decentralized certification directly influences this decision.

We also contribute to the literature on certifications and ecolabels. Increased consumer skepticism has driven demand for credible sustainability verification and transparency from brands (Leonidou and Skarmeas 2017). In response, firms use certifications to validate sustainability claims, with evidence showing consumers are willing to pay a premium for certified green products. Morone et al. (2021) finds that consumers in the European Union pay more for certified bio-based products than for conventional bio-based products. Similarly, consumers show higher willingness to pay for Energy Star certified products, and ecolabels like those from the U.S. Department of Agriculture (USDA) and the Marine Stewardship Council (MSC) also improve consumer willingness to pay (Houde 2022, Castka and Corbett 2016, Murali et al. 2019). However, the broader certification literature highlights fundamental challenges with these certification mechanisms: centralized certification intermediaries face tradeoffs between informativeness and deterring participation (Lizzeri 1999, Farhi et al. 2013), voluntary self-regulation can have ambiguous welfare effects (Maxwell et al. 2000, Lyon and Maxwell 2011), and uncertainty about label standards can erode certification value (Harbaugh et al. 2011). With the emergence of blockchain technology, decentralized consortia such as Aura offer a potential solution to these challenges, enabling firms to certify each other's products through tamper-proof peer-verification on distributed ledgers. There is empirical evidence documenting the increased willingness to pay associated with this new form of certification: Helliar et al. (2020) find that a permissioned blockchain implemented in the Italian wine industry adds an average price premium of 30% for verified wine and improves brand reputation by 9%. Lin et al. (2022) find that 37% of Chinese consumers are willing to pay a premium for U.S. beef that is traceable using blockchains. Despite evidence that consumers value decentralized certification, the technology faces limitations, most notably the challenge of ensuring data input integrity (Deloitte 2018, Chod et al. 2020). It remains unclear how this new form of peer-verification impacts firms' underlying sustainability decisions. Our paper addresses this question and shows that decentralized consortia, despite increasing firm profits, can paradoxically reduce sustainability incentives and harm consumer and social welfare.

Finally, we also contribute to a growing literature studying the operational impacts of blockchain; see, for example, Babich and Hilary (2020), Olsen and Tomlin (2020), Cui et al. (2019), Cui et al. (2020), Pun et al. (2021), Sumkin et al. (2021), Iyengar et al. (2022), and references therein. While many of these and other related studies focus on the potential profit advantages of this technology, our paper highlights non-trivial and perhaps unintended implications on firm green incentives.

3 Off-Consortium Game

As discussed in the introduction, our objective is to analyze and subsequently compare equilibrium outcomes and more specifically, green incentives, between two separate games: off-consortium and on-consortium. In this section, we focus first on the off-consortium game. We describe the model in Section 3.1, and present the equilibrium results of the game in Section 3.2.

3.1 Model Description

Consider an economy with $n \geq 2$ firms, indexed by $i = 1, \ldots, n$. As introduced in the setting of the Aura Consortium, these firms are in different markets and are not engaged in product market competition; instead, each firm has its own distinct customer base which is modeled as a continuum of heterogeneous rational consumers normalized to a mass of one. In Section 6.2, we relax this assumption and consider firms who compete in the product market. The firms are symmetric in the base model, and to simplify exposition, we take the perspective of a representative firm i as "the firm"; all statements apply to any other firm.

Firm i makes a sustainability production effort $e \in [0, 1]$ to produce 1 unit of a green product, at production cost $t(e)$. In line with sustainable practices and quality literature, $t(e)$ is positive, strictly increasing, and twice continuously differentiable, reflecting the rising costs associated with higher environmental quality (Plambeck and Taylor 2019, Murali et al. 2019, Corbett and DeCroix 2001). We focus on parameter values admitting unique interior equilibria in both games. Specifically, we assume the off-consortium and on-consortium profit functions each admit a unique critical point e^* in their respective interior pricing regions, at which the second-order condition holds: $t''(e^*) > v_l^2/(4\delta e^{*3})$ off-consortium and $t''(e^*) > (1-\alpha)^2 v_l^2/(4\delta e^{*3})$ on-consortium. We further assume that this interior critical point strictly dominates the corner solutions $e \in \{0, 1\}$. Firm i's effort level directly influences the probability of its product being 'green', e.g., meeting environmental standards throughout its entire supply chain. We set this probability to the effort level e (consequently, $1-e$ is the probability that the product is non-green). This probabilistic formulation reflects the reality that sustainability outcomes depend on factors beyond the firm's direct control, particularly in multi-tier supply chains where suppliers may evade compliance requirements (Plambeck and Taylor 2016). Thus, while greater sustainability effort (e.g., selecting responsible suppliers and establishing sourcing standards) increases the likelihood that the final product is genuinely green, it cannot eliminate the risk of non-compliance deeper in the supply chain.[2] The firm observes its chosen effort e but not the realized green/non-green status of its product absent external validation; the realized status is revealed only through the consortium's validation process described in Section 4.

There are two periods in the off-consortium game: The production period and the selling period. In the production period, firm i first chooses the sustainability effort level e, and the selling price p. In the selling period, products become available for consumers to purchase. Consumers have a low valuation $v_l > 0$ for non-green products, while green products have a valuation v that is uniformly distributed between v_l and v_h with an average consumer valuation of $\bar{v} \equiv \frac{v_h+v_l}{2}$, where $v_h > v_l$. We define $\delta \equiv \bar{v} - v_l = \frac{v_h-v_l}{2}$ for notational convenience. The realization of v for each consumer is private information that is not readily observable by the firms. This model of product valuation is common in papers that incorporate a premium for green products (Guo et al. 2016, Agrawal and Lee 2019, Gao and Souza 2022). Figure 1 depicts the timeline for the off-consortium game.

While consumers know their valuation for green and non-green products, the environmental quality of the products cannot be readily observed. Rather, consumers infer the quality and make purchasing decisions based on expectations. We assume that consumers form rational expectations

[2]For instance, Starbucks developed its C.A.F.E. Practices certification program to ensure ethical sourcing across its coffee supply chain. Yet farms participating in this program were found by Brazilian government inspectors to have violated environmental and labor standards (Business & Human Rights Resource Centre 2021). Similarly, Dior conducted regular environmental and social compliance audits across its supply chain, yet a subcontractor that had passed these audits was later found to be exploiting undocumented workers (Anzolin et al. 2024).

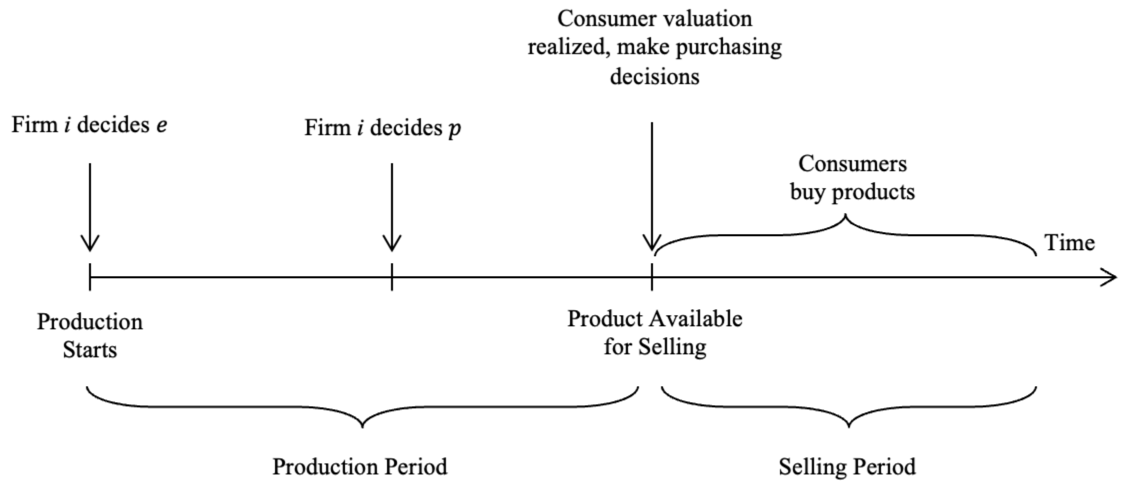


Figure 1: Sequence of Events for Off-Consortium Game

regarding the quality e (or, equivalently, the firm's effort level) and that they make purchasing decisions to maximize their expected utility (Baksi and Bose 2007, Karaer et al. 2017, Murali et al. 2019). Since the off-consortium game does not involve peer verification, there is no certification to represent the environmental quality of the product. Putting these elements together, a consumer with valuation v for green products, who purchases at price p has an expected utility

$$\mathcal{U}(v) = e\,v + (1-e)\,v_l - p, \tag{1}$$

where, as mentioned, e and $1-e$ are the probabilities of the product being green, and non-green, respectively. Consumers will purchase if the utility in (1) is nonnegative, and otherwise will not purchase. Formally,

Lemma 1 *(Consumer Purchasing Threshold Off-Consortium) In the off-consortium game, for given sustainability effort e and price p, there exists a unique threshold $\hat{v}$ on realized valuations such that consumers with $v \geq \hat{v}$ purchase, and those with $v < \hat{v}$ do not, where*

$$\hat{v} = \begin{cases} v_l & \text{if } e = 0 \text{ and } p \leq v_l, \\ v_h & \text{if } e = 0 \text{ and } p > v_l, \\ \frac{p-(1-e)v_l}{e} & \text{if } e > 0. \end{cases} \tag{2}$$

Define the expected demand from the firm's perspective as $D(e, p)$:

$$D(e,p) = \mathbb{P}(\mathcal{U}(v) \geq 0) = \min\left\{1,\, \max\left\{0,\, \frac{v_h - \hat{v}}{v_h - v_l}\right\}\right\}.$$

In the "off-consortium" game, firm i chooses e and p to maximize expected profit,

$$\Pi^0 = pD(e,p) - t(e), \tag{3}$$

and the optimization problem in the off-consortium game can be written as $\max_{e,p} \Pi^0(e,p)$.

3.2 Equilibrium Analysis

Next, we find the subgame perfect equilibrium for the off-consortium game. In the following proposition, we characterize the equilibrium sustainability production effort, e^*, and the equilibrium price, p^*, in the off-consortium game.

Proposition 1 *(Equilibrium Off-Consortium) In the off-consortium game, there exists a unique equilibrium such that:*

(i) the firm's sustainability effort e^* satisfies $e^* > 0$ and is uniquely determined by

$$t'(e^*) = \frac{4\,\delta^2\, e^{*2} - (\bar{v} - \delta)^2}{8\,\delta\, e^{*2}}, \quad \textit{and} \tag{4}$$

(ii) the firm's product price is $p^* = \frac{e^* v_h + (1-e^*) v_l}{2}$, and

(iii) the firm's equilibrium sustainability effort e^* and the product price p^* are increasing in $\bar{v}$.

Proposition 1 shows that in the off-consortium game, the firm's equilibrium sustainability effort and pricing decisions increase with consumers' average willingness to pay for green products, $\bar{v}$. As $\bar{v}$ rises, green products become more valuable to consumers, motivating the firm to put in more sustainability effort. This increased effort improves the likelihood that the firm's product is genuinely green, which allows the firm to charge a higher price and capture the greater willingness to pay from consumers.

4 On-Consortium Game

In this section, we analyze the on-consortium game. Here, firms have the option of forming a decentralized consortium to certify each other's products, much like the Aura Consortium mentioned in the introduction.

4.1 On-Consortium Model

The on-consortium game consists of three periods: (i) the *production period*, in which firms choose their sustainability production effort and submit product information to the consortium; (ii) the *certification and pricing period*, in which a firm is selected (via a mechanism to be detailed) to verify and record information onto the distributed ledgers. This validation/certification process, where the green status of the firm's product is required to be certified on the consortium, marks a major difference from the off-consortium game. After the certification outcome is realized, the firm then sets the price for its product; and (iii) the *selling period*, in which consumers observe their realized values and the certification outcome, and decide whether to purchase the product. Figure 2 depicts the timeline for the on-consortium game.

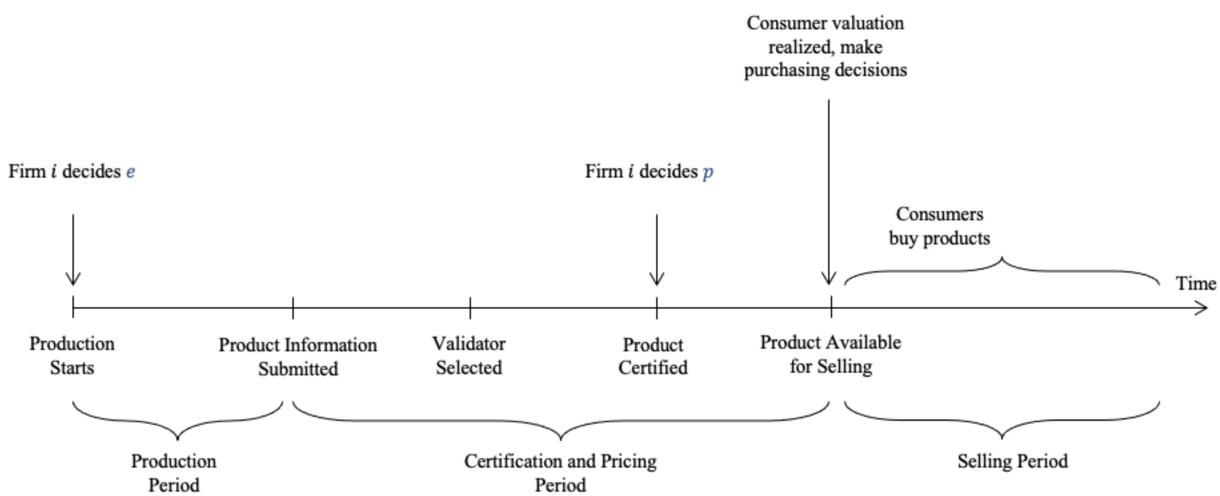


Figure 2: Sequence of Events for the On-Consortium Game

Production Period

As in the off-consortium game, the firm chooses sustainability effort e at cost $t(e)$. Following production, the firm submits the corresponding product information to the consortium so that the green status of the product can be validated and certified.

Validation and Pricing Period

In this period, one firm is randomly selected as the validator who is responsible for verifying the submitted information, certifying the green status of the product, and recording the green certificate on the ledger. We assume the focal firm cannot be selected as its own validator, consistent with consensus protocols for permissioned blockchain networks such as the Round Robin model.[3]

Given the complexities involved in understanding other firms' production processes and operations, the validator may face significant challenges in precisely verifying the green status of another firm's products. We thus assume that the validator can detect a non-green product produced by another firm with probability $\alpha \in (0, 1)$. To simplify, we normalize the cost of verification to zero but as an approximation, this could be embedded in α (higher verification costs would imply a lower detection probability).[4] We also assume that the validator cannot mistakenly (or strategically) label another firm's product as non-green when it is in fact green (no false negatives). This one-sided error structure is standard in the environmental disclosure literature (Lyon and Maxwell 2011).

After the certification outcome is realized, the firm observes the outcome and then sets the selling price for its product. We denote the price when the product is certified by p_h and the price for a non-certified product by p_l.

Selling Period

Consumers observe whether the product is certified along with the associated price.[5] This inference applies to products whose green claims are expected to be submitted for consortium verification; absence of verification is therefore informative in the model. Consumers consider possible scenarios when formulating their expected utilities from purchasing a certified or non-certified product. To

[3]Self-validation is a known concern with DL-based certification: a self-validating firm could in principle certify its own product regardless of true status, a manifestation of the "garbage in, garbage out" weakness of DL systems (Chod et al. 2020). Our main qualitative results are unchanged if self-validation is allowed; what changes is that self-certification (occurring with probability $\frac{1}{n}$) dilutes the credibility of the certified signal, with the effect vanishing as $n \to \infty$.

[4]In practice, there may be costs associated with verifying the sustainability status of another firm's products. In this model, we abstract from these costs by assuming that the detection ability is exogenously given. Settings characterized by lower detection ability may naturally correspond to higher verification costs, which could compel firms to adopt less rigorous verification measures. Conversely, settings with higher detection ability typically involve lower verification costs, allowing for more thorough and frequent checks of another firm's product sustainability.

[5]On decentralized consortia such as Aura, product information such as sustainability certifications is validated and recorded on the DL and made accessible to consumers through a QR code or NFC tag (see `https://auraconsortium.com/`). A product whose green claims have been verified displays this information, while a product whose claims are not verified will not have this information available. Our model's binary certified/non-certified outcome is a stylization of this informational difference. Non-certification in our model should thus be interpreted broadly as a product not being marketed with a verified sustainability claim on the consortium, rather than necessarily as a public failed-audit event.

that end, consumers considering firm i's product form rational expectations regarding the firm's sustainability effort, e. Specifically, consumers may be presented with a certified or a non-certified product and its associated price. If a product is not certified, they infer with certainty that it is non-green because no firm would falsely verify that a green product is non-green. Therefore, all consumers have the low valuation v_l for the product and pay the non-certified price p_l. Their utility in this case is:

$$\mathcal{U}_l(v) = v_l - p_l, \tag{5}$$

and they choose to purchase the product if (5) is non-negative. Hence, the expected demand for firm i's non-certified product, $D_l(p_l)$, is

$$D_l(p_l) = \int_{v_l}^{v_h} \frac{1}{v_h - v_l} \cdot \mathbb{1}_{\mathcal{U}_l(v) \geq 0}\, dv = \begin{cases} 1 & \text{if } p_l \leq v_l, \\ 0 & \text{otherwise.} \end{cases}$$

Alternatively, if the product is certified green, rational consumers consider the possibility that the product may have been mistakenly certified. In particular, although the consortium rules out self-validation, false certification may still occur if the validator fails to detect that the product is non-green. Thus, a consumer with valuation v for green products, who purchases a certified product from firm i at price p_h has utility

$$\mathcal{U}_h(v) = \mathbb{P}[\text{green} \mid \text{certified}] \cdot v + \mathbb{P}[\text{non-green} \mid \text{certified}] \cdot v_l - p_h. \tag{6}$$

To understand these probabilities, there are two possible cases to consider:

Case (i): firm i's product is green, which happens with probability:

$$\text{P[case (i)]} = \text{P[green]} = e.$$

Case (ii): firm i's product is non-green, but the validator fails to detect it. Since the validator can only detect a non-green product with probability α, the probability that case (ii) occurs is

$$\begin{aligned} \text{P[case (ii)]} &= \text{P[non-green]} \cdot \text{P[not detected]} \\ &= (1 - e) \cdot (1 - \alpha). \end{aligned}$$

Overall, the probability that firm i's product would be certified is

$$\begin{aligned} \mathrm{P}[\text{certified}] &= \mathrm{P}[\text{case (i)}] + \mathrm{P}[\text{case (ii)}] \\ &= e + (1-e)\cdot(1-\alpha). \end{aligned}$$

Thus, the consumer's utility, as outlined in (6), can be re-written as

$$\begin{aligned} \mathcal{U}_h(v) &= \frac{\mathrm{P}[\text{case (i)}]}{\mathrm{P}[\text{certified}]}\cdot v + \frac{\mathrm{P}[\text{case (ii)}]}{\mathrm{P}[\text{certified}]}\cdot v_l - p_h \\ &= \frac{e}{e+(1-e)(1-\alpha)}\cdot v + \frac{(1-e)(1-\alpha)}{e+(1-e)(1-\alpha)}\cdot v_l - p_h. \end{aligned} \tag{7}$$

A consumer chooses to purchase the certified product if her utility in (7) is non-negative. Formally,

Lemma 2 *(Consumer Purchasing Threshold On-Consortium) In the on-consortium game, for given sustainability effort e and certified price p_h, there exists a unique threshold $\hat{v}$ on realized valuations such that consumers with $v \geq \hat{v}$ purchase, and those with $v < \hat{v}$ do not, where*

$$\hat{v} = \begin{cases} v_l & \text{if } e = 0 \text{ and } p_h \leq v_l, \\ v_h & \text{if } e = 0 \text{ and } p_h > v_l, \\ \frac{p_h - \frac{(1-\alpha)(1-e)v_l}{e+(1-\alpha)(1-e)}}{\frac{e}{e+(1-\alpha)(1-e)}} & \text{if } e > 0. \end{cases} \tag{8}$$

Hence, the expected demand for firm i's certified product, $D_h(e, p_h)$, is given by

$$D_h(e,p_h) = \mathbb{P}(\mathcal{U}_h(v) \geq 0) = \min\left\{1,\, \max\left\{0,\, \frac{v_h - \hat{v}}{v_h - v_l}\right\}\right\}.$$

Firm i maximizes its expected profit, $\Pi^{\mathcal{C}}$:

$$\max_{e,p_h,p_l} \quad \Bigg\{ \underbrace{[e + (1-e)\,(1-\alpha)]\,p_h\,D_h}_{\text{Expected revenue if certified}} + \underbrace{(1-e)\,\alpha\,p_l\,D_l}_{\text{Expected revenue if non-certified}} - t(e) \Bigg\} \tag{9}$$

where D_h and D_l are the expected demand for firm i's certified and non-certified product, respectively.

4.2 Equilibrium Analysis

The following proposition characterizes firm i's equilibrium results in the on-consortium game.

Proposition 2 *(Equilibrium On-Consortium) In the on-consortium game, there exists a unique equilibrium such that:*

(i) the sustainability effort e^* satisfies $e^* > 0$ and is uniquely determined by

$$t'(e^*) = \frac{e^{*2}((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^{*2}}, \tag{10}$$

(ii) if the product is certified, then the firm's equilibrium price is $p_h{}^* = \frac{e^* v_h + (1-\alpha)(1-e^*)v_l}{2+2\alpha(-1+e^*)}$; if the product is non-certified, then the firm's equilibrium price is $p_l{}^* = v_l$, and

(iii) the firm's equilibrium sustainability effort e^* and the certified price $p_h{}^*$ are increasing in $\bar{v}$.

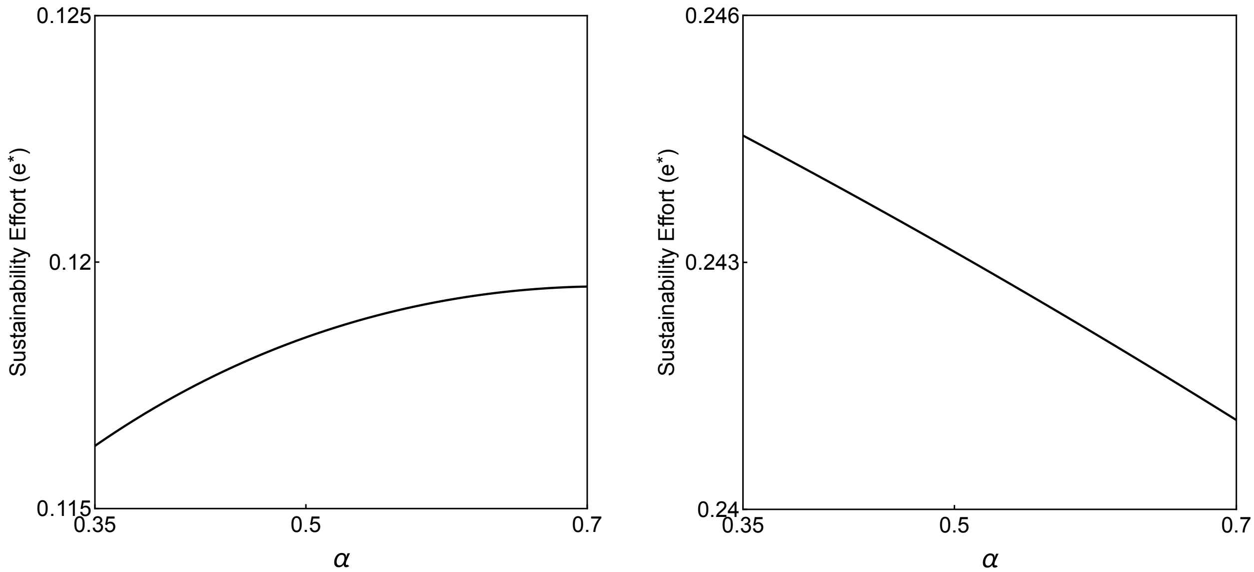
Proposition 2 shows that in the on-consortium game, the firm's equilibrium sustainability effort and the price it can charge when the product is certified increase with the average consumer willingness to pay for green products, $\bar{v}$. The underlying rationale is similar to that in the off-consortium case: as $\bar{v}$ rises, green products become more valuable to consumers, motivating the firm to invest more in sustainability effort. This increased sustainability effort improves the likelihood that the firm's certified product is genuinely green, enhancing its perceived credibility among consumers and enabling the firm to charge a higher price if the product is certified. However, unlike the off-consortium case, where the product's true green status is never revealed to consumers, a key distinction in the on-consortium case is that if the product is not certified, consumers know with certainty that it is non-green. As a result, the firm can only set a lower price corresponding to consumers' low valuation for non-green products, v_l.

Proposition 3 *(Comparative Statics of On-Consortium Game). There exists a unique threshold $\hat{e}^\alpha \equiv \sqrt{\frac{(1-\alpha)\,v_l}{2\delta - \alpha v_l}}$ such that the on-consortium equilibrium sustainability effort e^* is increasing in α if $e^* < \hat{e}^\alpha$, and decreasing in α if $e^* > \hat{e}^\alpha$.*

Proposition 3 shows that the on-consortium equilibrium sustainability effort e^* is not monotone in the validator's verification accuracy α. The reason is that a higher α has two opposing effects on the firm's incentive to invest in sustainability. On one hand, a stricter validator makes certification more credible: consumers in the certified state are more confident that the product is genuinely green, and the firm earns a higher price when certified. This raises the firm's payoff per certification, pulling the firm to invest more in sustainability. On the other hand, a stricter validator also raises the risk that the firm's product is caught when it is non-green: the product then sells at the low price v_l, leaving the firm without the certified-state premium. This pulls the firm to invest less.

Figure 3: Impact of validator verification accuracy on the Equilibrium Sustainability Effort in which $v_l = 0.05, t(e) = e^2$.

(a) $\bar{v} = 0.57$ (b) $\bar{v} = 1.05$



When the firm's equilibrium effort e^* is below the threshold $\hat{e}^\alpha$ (equivalently, when the average consumer willingness to pay $\bar{v}$ is low), the certified-state premium has substantial room to grow with stricter validation. The premium force dominates, and higher α raises e^*. When e^* is above $\hat{e}^\alpha$ (equivalently, when $\bar{v}$ is high), the certified-state premium is already near its ceiling, so stricter validation can barely raise it further. The risk force dominates, and higher α reduces e^*. We illustrate these results in Figure 3.

5 Implications for Green Incentives and Technology Adoption

Given the equilibrium analysis on and off-consortium, in this section, we examine how joining a consortium can affect firm green incentives and welfare outcomes. We use superscript 0 to denote off-consortium equilibrium quantities, and superscript $\mathcal{C}$ to denote on-consortium equilibrium quantities.

5.1 Impact of Consortium on the Firm

Proposition 4 compares the equilibrium outcomes for the off-consortium and on-consortium games.

Proposition 4 *(Comparison of Equilibrium Product Prices, Sustainability Effort, and Profit).*

The equilibrium outcomes of the on-consortium and off-consortium games satisfy:

(i) Prices: $p_h^*(e) \geq p^*(e) \geq p_l^*$ *for any common effort* $e \in (0, 1)$.

(ii) Sustainability effort levels: there exists a unique threshold $\hat{e} \equiv \sqrt{\frac{(2-\alpha)\, v_l}{4\delta - \alpha v_l}}$ *such that* $e^{\mathcal{C}*} > e^{0*}$ *if* $e^{0*} < \hat{e}$, *and* $e^{\mathcal{C}*} < e^{0*}$ *if* $e^{0*} > \hat{e}$.

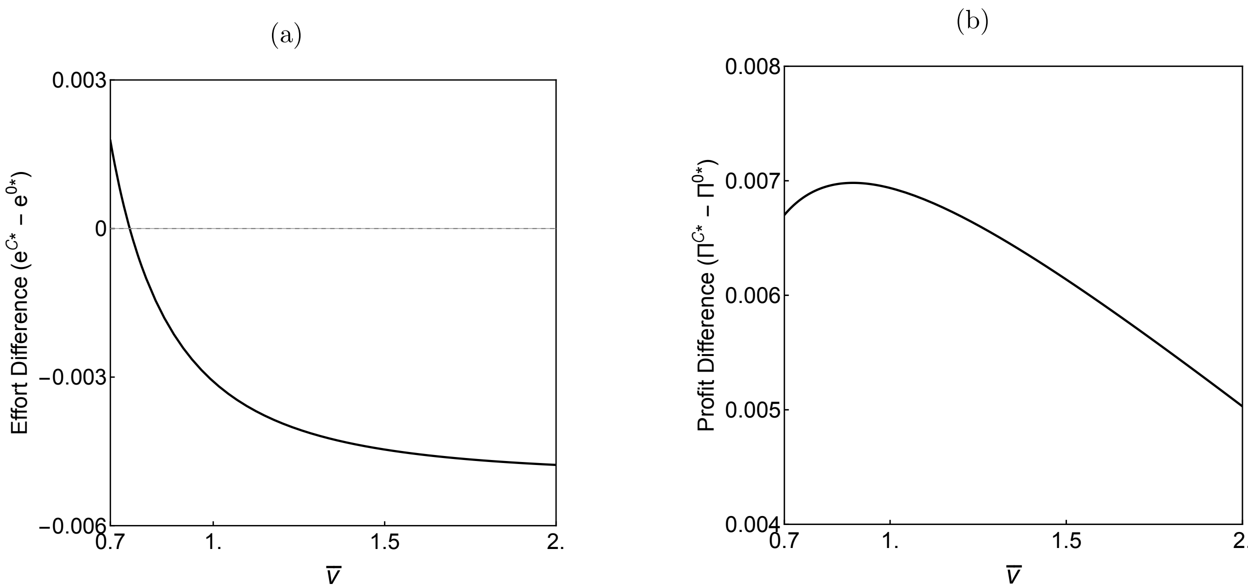
(iii) Profits: $\Pi^{\mathcal{C}*} \geq \Pi^{0*}$.

Proposition 4(i) shows that, at any sustainability effort, the firm charges a price premium on the consortium for certified products relative to the off-consortium pooled price. This is because certification improves informativeness for consumers: conditional on being certified, consumers are more confident that the product is genuinely green, which increases their willingness to pay and allows the firm to charge a higher price. This is consistent with real-world examples. For example, some Italian wine producers are charging an average of 30% more for consortium-verified wine, and consumer surveys indicate a willingness to pay more for consortium-traced beef (Helliar et al. 2020, Lin et al. 2022). Similarly, in the diamond industry, consumers report willingness to pay 10 to 20% more for diamonds with blockchain-verified sustainable provenance (De Beers Group 2021). At the same time, Proposition 4(i) also shows that if the product is not certified in the on-consortium case, then the firm charges a lower price than off the consortium. Since there are no false negatives, consumers can infer with certainty that the product is not green, so the firm can only price the product at a lower level consistent with consumers' low valuation.

Despite the constraint to charge a lower price when a non-green product fails certification, Proposition 4(iii) shows that the firm always benefits from joining the consortium. To see why, consider a firm that joins the consortium but maintains the same sustainability effort as off the consortium, $e = e^{0*}$. Even at this fixed effort level, the firm's expected profit is weakly higher on the consortium than off, because the certification outcome allows the firm to separate prices by state rather than charge a single pooled price, which creates two advantages: (1) extracting more revenue per buyer in the certified state, and (2) reaching a broader set of consumers in the non-certified state. In the certified state, the firm can charge a higher price without sacrificing demand because consumers are more confident that the product is genuinely green. In the non-certified state, the firm prices low but reaches a broader set of consumers. As a result, the firm's expected revenue is always weakly higher on the consortium. The firm can further improve its profit by re-optimizing its sustainability effort on the consortium. Taken together, these effects drive higher profits on the consortium.

Most interestingly, despite the improvements in profit, Proposition 4(ii) shows that the consortium has an ambiguous effect on the firm's sustainability effort. The reason is that joining the

Figure 4: Impact of Consortium on Sustainability Effort and Firm Profit, as a Function of $\bar{v}$, in which $v_l = 0.05, \alpha = 0.4, t(e) = e^2$. Panel (a) plots the effort difference $e^{\mathcal{C}*} - e^{0*}$ and Panel (b) plots the profit difference $\Pi^{\mathcal{C}*} - \Pi^{0*}$.



consortium creates two opposing forces on the firm's incentive to invest in sustainability. On one hand, the consortium enables the firm to charge a price premium relative to the off-consortium pooled price when the product is certified. Since the likelihood of certification increases with sustainability effort, this pulls the firm to invest more in sustainability than it would off the consortium. On the other hand, if the product fails to be certified, the firm has to sell at the low price v_l, which is below the off-consortium pooled price. The firm faces the risk that its effort doesn't lead to certification, rendering the investment "wasted". This pulls the firm to invest less.

When the off-consortium baseline effort e^{0*} is low (equivalently, when the average consumer willingness to pay $\bar{v}$ is low), the certified-state premium has substantial room to grow with effort. The premium force dominates, and the firm invests more on the consortium ($e^{\mathcal{C}*} > e^{0*}$). When e^{0*} is high (equivalently, when $\bar{v}$ is high), the certified-state premium is already near its ceiling, leaving effort with little room to raise it further. The risk force dominates, and the firm invests less on the consortium ($e^{\mathcal{C}*} < e^{0*}$).

We illustrate the results of Proposition 4 in Figure 4 for a representative case in which $v_l = 0.05$, $\alpha = 0.4$, $t(e) = e^2$. The horizontal axis in both panels is the average consumer willingness to pay $\bar{v}$. Panel (a) plots the effort difference $e^{\mathcal{C}*} - e^{0*}$: the consortium raises sustainability effort when $\bar{v}$ is sufficiently low and reduces it when $\bar{v}$ is sufficiently high. Panel (b) plots the profit difference and shows that the consortium always improves firm profit.

Overall, the insights from Proposition 4 highlight the mixed impact that consortium adoption has on the firm's equilibrium actions: while it consistently enhances profitability, it may induce investment in lower sustainability efforts.

5.2 Welfare Implications

Our results show that firms always benefit from joining the consortium. However, given the differences in prices and effort levels on/off the consortium, what are the implications on consumer and social welfare? We consider the welfare implications of consortium adoption in this section. In the off-consortium case, consumer surplus, CS^0, is defined as:

$$CS^0 = \int_{v_l}^{v_h} \mathcal{U}(v) \cdot \frac{1}{v_h - v_l} \cdot \mathbb{1}_{\mathcal{U}(v)\geq 0}\, dv,$$

In the on-consortium game, the total consumer surplus, $CS^{\mathcal{C}}$, consists of the expected consumer surplus from a certified product, $CS_h^{\mathcal{C}}$, and the expected consumer surplus from a non-certified product, $CS_l^{\mathcal{C}}$. They are defined as follows:

$$\begin{aligned}
CS_h^{\mathcal{C}} &= \mathrm{P}[\text{certified}] \cdot \int_{v_l}^{v_h} \mathbb{E}\mathcal{U}_h(v) \cdot \frac{1}{v_h - v_l} \cdot \mathbb{1}_{\mathbb{E}\mathcal{U}_h(v)\geq 0}\, dv, \\
CS_l^{\mathcal{C}} &= (1 - \mathrm{P}[\text{certified}]) \cdot \int_{v_l}^{v_h} \mathcal{U}_l(v) \cdot \frac{1}{v_h - v_l} \cdot \mathbb{1}_{\mathcal{U}_l(v)\geq 0}\, dv, \\
CS^{\mathcal{C}} &= CS_h^{\mathcal{C}} + CS_l^{\mathcal{C}},
\end{aligned}$$

The social welfare in both the off-consortium and on-consortium game is defined as the sum of firm's profits and consumer surplus, i.e., $W^i = \Pi^i + CS^i$ where $i \in \{0, \mathcal{C}\}$. In the following proposition, we compare the consumer surplus and social welfare in the on-consortium and the off-consortium game.

Proposition 5 *(Comparison of Consumer Surplus and Social Welfare) The equilibrium outcomes of the on-consortium and off-consortium games satisfy:*

(i) $CS^{\mathcal{C}*} < CS^{0*}$ *whenever* $e^{\mathcal{C}*} < e^{0*}$*. If* $e^{\mathcal{C}*} \geq e^{0*}$*, then* $CS^{\mathcal{C}*} < CS^{0*}$ *if and only if*

$$\frac{\left((1-\alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*}\right)^2}{e^{\mathcal{C}*}} < \frac{(v_l + 2\delta\, e^{0*})^2}{e^{0*}}. \tag{11}$$

(ii) $W^{\mathcal{C}*} > W^{0*}$ *whenever* $CS^{\mathcal{C}*} \geq CS^{0*}$. *If* $e^{\mathcal{C}*} < e^{0*}$, *then* $W^{\mathcal{C}*} < W^{0*}$ *if and only if*

$$\frac{3\,(v_l + 2\delta\, e^{0*})^2}{16\delta\, e^{0*}} - \frac{3\left((1-\alpha)\,v_l + (\alpha v_l + 2\delta)\,e^{\mathcal{C}*}\right)^2}{16\delta\, e^{\mathcal{C}*}} \;>\; \alpha(1-e^{\mathcal{C}*})\,v_l + t(e^{0*}) - t(e^{\mathcal{C}*}). \quad (12)$$

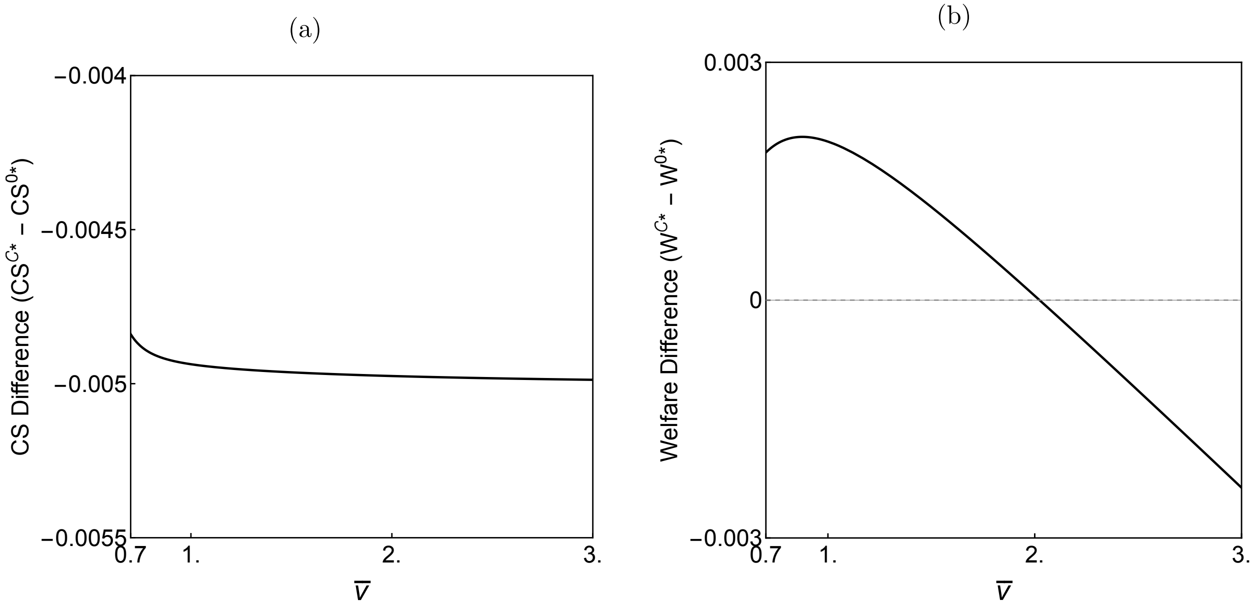
Proposition 5(i) shows that the consortium affects consumer surplus through two channels. The first is a *state-contingent pricing channel*: at any fixed effort, the consortium reduces consumer surplus because certification lets the firm condition its price on the certification outcome, charging a higher price when the product is certified and pricing at v_l to extract all surplus when it is not. The second is an *effort channel*: higher equilibrium effort raises consumer surplus by making certified products both more likely and more credible. When the consortium reduces effort, both channels work against consumers, and consumer surplus unambiguously falls. When the consortium raises effort, the effort channel pulls consumer surplus up while the state-contingent pricing channel pulls it down; consumer surplus falls if and only if the latter dominates, as captured by condition (11).

Proposition 5(ii) shows that social welfare can similarly fall in equilibrium. Social welfare is the sum of the firm's profit and consumer surplus, and the firm always benefits from joining the consortium (Proposition 4(iii)). When the consortium raises consumer surplus, both components rise and welfare unambiguously increases. When the consortium reduces consumer surplus, however, the firm's profit gain partially offsets the consumer-surplus loss; welfare falls only when the consumer-surplus loss exceeds the profit gain, as captured by condition (12). This finding carries an unintuitive policy implication: even though the firm strictly benefits from joining the consortium, society can be worse off in markets where the consortium reduces sustainability incentives.

We illustrate the consortium's effects on consumer surplus and social welfare in Figure 5 for a representative case under quadratic effort cost. The horizontal axis in both panels is the average consumer willingness to pay $\bar{v}$. Panel (a) shows that the consortium reduces consumer surplus across the range of $\bar{v}$, as the state-contingent pricing channel dominates the effort channel even when the consortium raises effort. Panel (b) shows that the consortium reduces social welfare when $\bar{v}$ is sufficiently high, as the consumer-surplus loss eventually outweighs the firm's profit gain.

Our results also carry a direct environmental interpretation. In our model, the firm's sustainability effort reflects the unconditional probability that a product is genuinely green. Proposition 4(ii) therefore implies that when the off-consortium effort exceeds the threshold $\hat{e}$ (equivalently, when consumers' average willingness to pay for green products is sufficiently high), consortium adoption reduces the expected environmental quality of products compared to the off-consortium benchmark. We further analyze this environmental dimension in Section 6.1, where we extend the

Figure 5: Impact of Consortium on Consumer Surplus and Social Welfare, as a Function of $\bar{v}$, in which $v_l = 0.05, \alpha = 0.4, t(e) = e^2$. Panel (a) plots the consumer-surplus difference $CS^{\mathcal{C}*} - CS^{0*}$ and Panel (b) plots the welfare difference $W^{\mathcal{C}*} - W^{0*}$.



model to incorporate an intrinsic sustainability preference into the firm's objective function.

Overall, our findings in Propositions 4 and 5 indicate that while consortia benefit firms, they may act as a double-edged sword that harms consumer and social welfare.

6 Extensions & Robustness

6.1 Sustainability Preferences

The main model assumes that firms' sustainability decisions are driven purely by profit maximization. In practice, firms may also have an intrinsic preference for sustainability, driven by corporate values, investor pressure, or anticipated regulatory penalties for non-green outcomes. In this section, we extend the model to incorporate such preferences. Specifically, we modify the firm's profit function by introducing a penalty $\gamma(1 - e)$ that the firm incurs in proportion to the probability that its product is non-green. The parameter $\gamma \geq 0$ captures the strength of the firm's sustainability preference; when $\gamma = 0$, the model reduces to our baseline. The modified off-consortium and on-consortium profit functions become:

$$\Pi^0 = p \cdot D - t(e) - \gamma(1-e),$$
$$\Pi^{\mathcal{C}} = [e + (1-e)(1-\alpha)]\, p_h\, D_h + (1-e)\alpha\, p_l\, D_l - t(e) - \gamma(1-e).$$

Since the penalty depends only on the firm's sustainability effort, the firm's optimal pricing decisions for a given effort level are unchanged; only the equilibrium effort is directly affected. In the following proposition, we characterize the impact of the sustainability preference on the equilibrium sustainability effort.

Proposition 6 *(Impact of Sustainability Preferences) When firms incorporate intrinsic sustainability preferences into their objective function ($\gamma > 0$):*

(i) The equilibrium sustainability effort levels e^{0*} and $e^{\mathcal{C}*}$ are strictly increasing in γ.

(ii) $e^{\mathcal{C}*} > e^{0*}$ if $e^{0*} < \hat{e}$, and $e^{\mathcal{C}*} < e^{0*}$ if $e^{0*} > \hat{e}$, where $\hat{e}$ is the threshold defined in Proposition 4(ii).

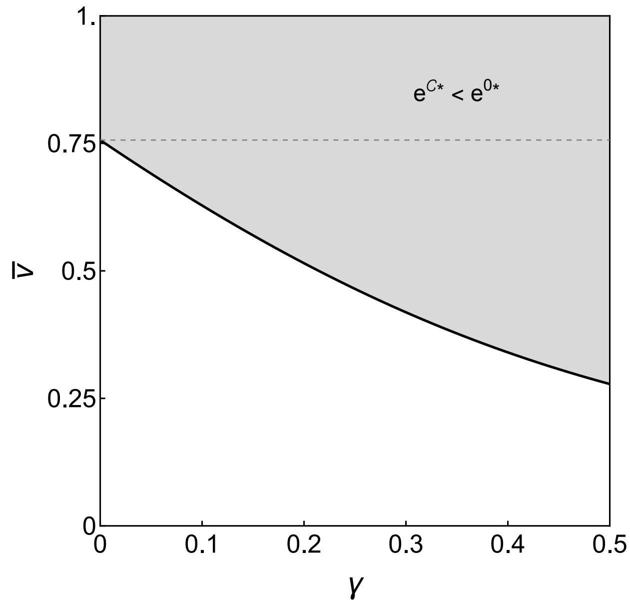
(iii) The parameter region in which the consortium reduces sustainability effort ($e^{0*} > \hat{e}$) expands as γ rises.

Proposition 6(i) confirms that the sustainability penalty motivates higher effort in both settings, as firms internalize the cost of producing non-green products. Part (ii) shows that our main sustainability result from Proposition 4(ii) remains qualitatively robust: the same $\hat{e}$ threshold continues to govern the effort comparison, with the consortium reducing effort when the off-consortium baseline e^{0*} exceeds $\hat{e}$. Part (iii) follows because $\hat{e}$ does not depend on γ, while e^{0*} is increasing in γ. Thus, stronger sustainability preferences move more parameter values into the high-baseline-effort region $e^{0*} > \hat{e}$, where consortium adoption reduces effort. This result suggests that the consortium's negative impact on sustainability incentives is driven by the structural features of decentralized certification rather than by firms' lack of commitment to sustainability. We illustrate the expanding regime in Figure 6.

6.2 Competition

The main model assumes that firms are in different markets and are not engaged in product market competition; each firm has its own distinct customer base. In this section, we relax this assumption

Figure 6: Impact of sustainability preference γ on the parameter region in which the consortium reduces sustainability effort, in which $v_l = 0.05, \alpha = 0.4, t(e) = e^2$. The solid curve plots the threshold value of $\bar{v}$ at which $e^{0^*} = \hat{e}$ as a function of γ. The shaded region above the curve corresponds to parameter values where $e^{0^*} > \hat{e}$, i.e., where the consortium reduces sustainability effort (Proposition 4(ii)). The dashed horizontal line indicates the baseline threshold at $\gamma = 0$.



and consider an extension in which two consortium members compete in the product market with differentiated products.

To capture competition, we adopt a Hotelling duopoly model adjusted to our setting. Two firms, indexed by $i \in \{1, 2\}$, are located at the endpoints of a unit interval, with firm 1 at 0 and firm 2 at 1. Consumers are indexed by (x, z), where $x \sim U[0, 1]$ is the consumer's ideal-point location on the line and $z \sim U[0, 2\delta]$ is the consumer's incremental valuation for a green product over a non-green product. Thus, if a consumer's valuation for a non-green product is v_l, her valuation for a green product is $v_l + z$, with $\mathbb{E}[z] = \delta$. The two dimensions of consumer heterogeneity are independent. Let q_i denote consumers' posterior belief that firm i's product is green. A consumer at location x with green-valuation realization z who purchases firm i's product at price p_i has utility:

$$\mathcal{U}_i(x, z) = \underline{v} + v_l + q_i z - p_i - \tau d_i(x), \tag{13}$$

where $d_1(x) = x$, $d_2(x) = 1 - x$, $\tau > 0$ is the Hotelling transportation cost that captures the intensity of product market competition, and $\underline{v}$ is a baseline market value that ensures participation. Intuitively, a lower τ corresponds to stronger competition, as consumers are less attached to their preferred firm and switch more readily on price or perceived quality. We focus on the covered-market case with the indifferent consumer in the interior of $[0,1]$ for all relevant states; sufficient conditions are $\tau > 4\delta/3$ and $\underline{v} + v_l > 3\tau/2$. For a realized pair of posterior beliefs (q_i, q_j) and prices (p_i, p_j), firm i's demand is

$$D_i(q_i, q_j, p_i, p_j) = \frac{1}{2} + \frac{\delta(q_i - q_j) - p_i + p_j}{2\tau}. \tag{14}$$

For analytical tractability, we maintain the quadratic cost specification $t(e) = e^2$ throughout this section.

Off the consortium, consumers do not observe any certification signal. Following the main model, they form rational expectations about each firm's sustainability effort, so $q_i = e_i$. Firm i chooses effort e_i and price p_i to maximize expected profit:

$$\max_{e_i, p_i} \left\{ p_i D_i(e_i, e_j, p_i, p_j) - t(e_i) \right\}, \tag{15}$$

where D_i is firm i's demand under (13) in the simultaneous-move pricing equilibrium given the efforts (e_i, e_j).

On the consortium, each firm's certification outcome is publicly observed before prices are set. The probability that firm i's product is certified is

$$s(e_i) = e_i + (1 - e_i)(1 - \alpha) = 1 - \alpha + \alpha e_i, \tag{16}$$

and the complementary probability that it is not certified is $\alpha(1 - e_i)$. Conditional on certification, consumers update their posterior that firm i's product is green to

$$q_h(e_i) = \frac{e_i}{s(e_i)}, \tag{17}$$

while consumers infer with certainty that a non-certified product is non-green, so $q_l = 0$. Let $Q_i \in \{q_h(e_i), 0\}$ denote the random posterior belief about firm i's product induced by certification: $Q_i = q_h(e_i)$ with probability $s(e_i)$ and $Q_i = 0$ with probability $\alpha(1 - e_i)$. The mean of this posterior is $\mathbb{E}[Q_i] = e_i$, and its variance is

$$V(e_i) \equiv \mathrm{Var}(Q_i) = \frac{\alpha(1 - e_i)\, e_i^2}{1 - \alpha + \alpha e_i}. \tag{18}$$

We assume certification outcomes are independent across firms conditional on efforts, which results in four possible realized pricing states:

$$(h,h), \quad (h,l), \quad (l,h), \quad (l,l), \tag{19}$$

where the first letter denotes firm 1's certification status and the second firm 2's. Firm i chooses effort e_i and a state-contingent price schedule $\{p_i^{xy}\}_{x,y\in\{h,l\}}$ to maximize expected profit:

$$\max_{e_i,\{p_i^{xy}\}} \left\{ \sum_{x,y\in\{h,l\}} \Pr(x,y \mid e_i,e_j) \cdot p_i^{xy} D_i^{xy}(\cdot) - t(e_i) \right\}, \tag{20}$$

where $D_i^{xy}(\cdot)$ is firm i's demand under (13) in the simultaneous-move pricing equilibrium of state (x,y) with posteriors $(q_x(e_i), q_y(e_j))$ and prices (p_i^{xy}, p_j^{xy}), and the state probabilities are determined by (e_i, e_j, α).

We characterize the equilibrium outcomes of the off-consortium and on-consortium Hotelling games in Propositions A.1 and A.2, respectively (see appendix). Here, we compare the equilibrium outcomes of the two games.

Proposition 7 *(Comparison of Equilibrium Prices, Sustainability Effort, and Profit under Hotelling Competition). The equilibrium outcomes of the on-consortium and off-consortium Hotelling games satisfy:*

(i) Prices: Let $\bar{p}_h^*$ and $\bar{p}_l^*$ denote firm i's expected equilibrium price conditional on being certified and non-certified, respectively, and p^{0*} be the off-consortium price. Then $p^{0*} = \tau$ and $\bar{p}_h^* \geq p^{0*} \geq \bar{p}_l^*$. A certified firm charges a premium only when its rival is not certified. Conditional on firm i's own certification status, $\bar{p}_h^* = \tau + \frac{\delta}{3} \cdot \frac{\alpha(1-e^{\mathcal{C}*})e^{\mathcal{C}*}}{1-\alpha+\alpha e^{\mathcal{C}*}}$ and $\bar{p}_l^* = \tau - \frac{\delta e^{\mathcal{C}*}}{3}$. The unconditional expected price, $\mathbb{E}[p_i^{\mathcal{C}*}]$, satisfies $\mathbb{E}[p_i^{\mathcal{C}*}] = p^{0*}$.

(ii) Sustainability effort: There exists a unique threshold $\widehat{e}(\alpha) \in [1/2, 2/3]$ such that $e^{\mathcal{C}*} > e^{0*}$ if $e^{0*} < \widehat{e}(\alpha)$, and $e^{\mathcal{C}*} < e^{0*}$ if $e^{0*} > \widehat{e}(\alpha)$.

(iii) Profits: $\Pi^{\mathcal{C}*} \geq \Pi^{0*}$, with strict inequality whenever $\alpha > 0$ and $e^{\mathcal{C}*} > 0$.

Proposition 7(i) shows that, under competition, certification creates state-contingent price dispersion rather than a uniform premium. When both firms share the same certification status, neither holds a credibility advantage and both charge τ which is the same as the off-consortium price. The certification premium emerges only in mixed states: in state (h,l), consumers know firm j's product is non-green with certainty (since certification has no false negatives) while firm

i's is likely to be green, so firm i charges above τ and firm j below. Conditional on a firm's own certification status, its expected price thus rises above τ when certified and falls below τ when not, consistent with the price pattern in the base model. Unconditionally, however, the expected price remains τ: gains in state (h, l) are offset by losses in state (l, h).

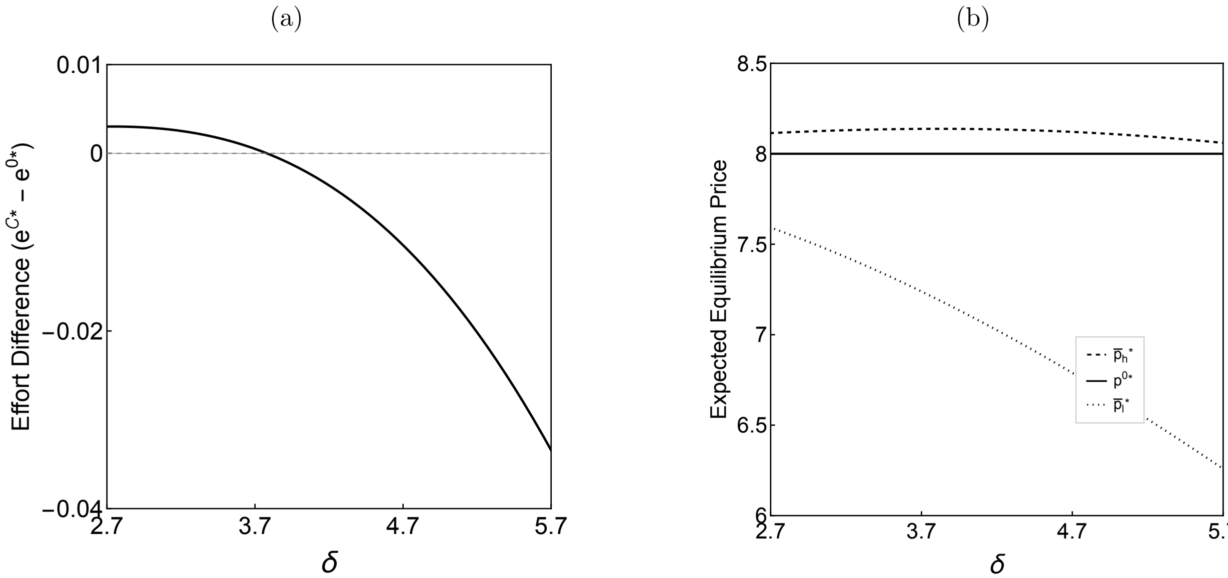
Proposition 7(iii) shows that the firm always strictly benefits from joining the consortium. To see why, consider a firm that joins but maintains the same effort as off the consortium, $e = e^{0*}$. Even at this effort, the firm earns higher expected profit because certification creates posterior dispersion across firms: in states where the firm has a credibility advantage, both its price and market share rise; in states where it has a disadvantage, both fall. Since revenue is the product of price and demand, this co-movement amplifies the firm's gains relative to its losses, making its expected revenue strictly higher on the consortium. The firm can therefore at least replicate its off-consortium profit and improve further by re-optimizing its effort, consistent with the base model.

Although the firm always benefits in terms of profit, Proposition 7(ii) shows that the consortium can either raise or lower sustainability effort, depending on the off-consortium baseline effort. Certification creates a mean-preserving spread in consumers' posterior beliefs: the average is unchanged, but realized posteriors are high when certified and zero when not. Because wider dispersion drives the credibility-advantage premium in part (i), the marginal value of effort depends on whether it enlarges or shrinks this dispersion. At low baseline effort, additional effort raises consumers' confidence in certified products, widening the dispersion and raising the marginal return. At high baseline effort, certification is nearly guaranteed, so additional effort reduces the small weight on non-certified outcomes, shrinking the dispersion and lowering the marginal return. Hence, when $e^{0*} > \widehat{e}(\alpha)$, consortium adoption reduces sustainability effort. Consistent with the base model, the consortium can weaken sustainability incentives even though firms are strictly better off.

We illustrate parts (i) and (ii) of Proposition 7 in Figure 7 for a representative case in which $v_l = 0.05$, $\alpha = 0.4$, $\tau = 8$, $t(e) = e^2$. Panel (a) shows that the consortium raises sustainability effort when $\delta < \delta^*$ and reduces it when $\delta > \delta^*$ (since $e^{0*} = \delta/6$ under quadratic costs, the threshold from Proposition 7(ii) translates to $\delta^* = 6\widehat{e}(\alpha)$). Panel (b) shows the expected equilibrium prices conditional on certification status: the certified-firm expected price $\bar{p}_h^*$ stays above τ while the non-certified-firm expected price $\bar{p}_l^*$ falls below τ, with the gap to τ widening as δ grows.

Proposition 8 *(Comparison of Consumer Surplus and Social Welfare under Hotelling Competition). The equilibrium outcomes of the on-consortium and off-consortium Hotelling games satisfy:*

Figure 7: Comparison of Sustainability Effort and Expected Equilibrium Prices under Hotelling Competition in which $v_l = 0.05, \alpha = 0.4, \tau = 8, t(e) = e^2$.



(i) Consumer surplus: $CS^{\mathcal{C}*} \geq CS^{0*}$ whenever $e^{\mathcal{C}*} \geq e^{0*}$. If $e^{\mathcal{C}*} < e^{0*}$, then $CS^{\mathcal{C}*} < CS^{0*}$ if and only if

$$\delta \left(e^{0*} - e^{\mathcal{C}*}\right) > \frac{2\delta^2}{9\tau} V(e^{\mathcal{C}*}). \tag{21}$$

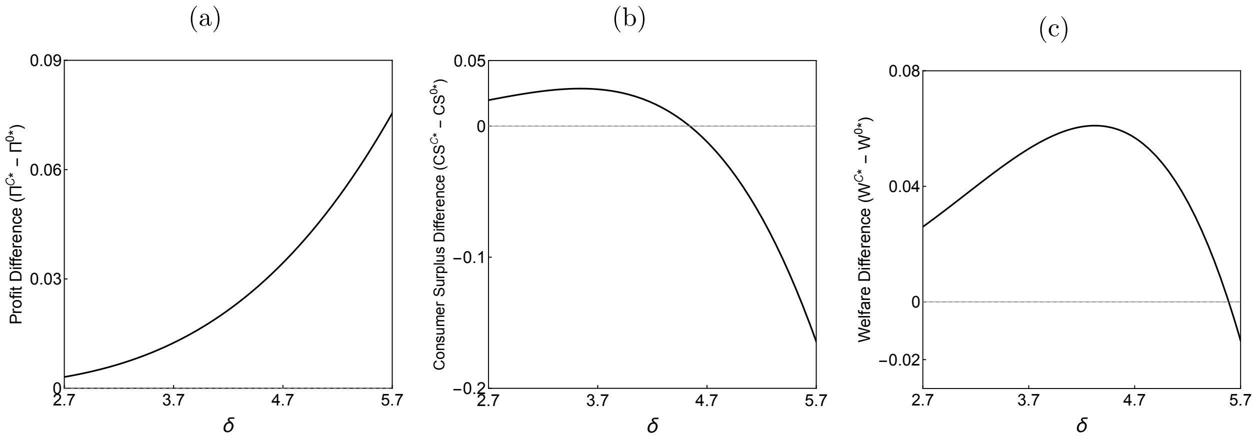
(ii) Social welfare: $W^{\mathcal{C}*} \geq W^{0*}$ whenever $e^{\mathcal{C}*} \geq e^{0*}$. If $e^{\mathcal{C}*} < e^{0*}$, then $W^{\mathcal{C}*} < W^{0*}$ if and only if

$$\delta \left(e^{0*} - e^{\mathcal{C}*}\right) > \frac{4\delta^2}{9\tau} V(e^{\mathcal{C}*}) + 2\left[(e^{0*})^2 - (e^{\mathcal{C}*})^2\right]. \tag{22}$$

Proposition 8(i) shows that the consortium has mixed effects on consumer surplus. At any fixed effort, certification raises consumer surplus because consumers can shift their purchases based on whether a product is certified. At equilibrium, however, consumer surplus also depends on the firm's effort, since higher effort raises the probability that consumers' purchases are genuinely green. When the consortium raises effort, both effects point in the same direction and consumer surplus rises. When it reduces effort, consumer surplus falls only if the lower green probability outweighs the information benefit. Consistent with the base model, the consortium can reduce consumer surplus when it sufficiently lowers sustainability effort.

Proposition 8(ii) shows that social welfare can similarly fall in equilibrium. Social welfare is the sum of consumer surplus and both firms' profits, and firms always profit from joining the consortium (Proposition 7(iii)). When the consortium raises effort, both consumer surplus and profits rise, so welfare unambiguously increases. When it reduces effort, the firms' total profit

Figure 8: Impact of Consortium on Firm Profit, Consumer Surplus, and Social Welfare under Hotelling Competition in which $v_l = 0.05, \alpha = 0.4, \tau = 8, t(e) = e^2$.



gain partially offsets the consumer-surplus loss; welfare falls only when this loss exceeds the gain. This finding has an unintuitive policy implication: even though firms strictly benefit from joining the consortium, society can be worse off in markets where the consortium reduces sustainability incentives.

We illustrate the consortium's impact on firm profit (Proposition 7(iii)) together with consumer surplus and social welfare (Proposition 8) in Figure 8. Panel (a) shows that the consortium uniformly raises firm profit, with the gain growing in δ. Panel (b) shows that the consortium can reduce consumer surplus when δ is sufficiently large, even though it raises consumer surplus at smaller δ (the sign change reflects the condition in Proposition 8(i), which translates to δ via $e^{0*} = \delta/6$). Panel (c) shows the analogous result for social welfare: the consortium raises welfare for moderate δ but reduces it for sufficiently large δ, confirming that the consortium can harm society as a whole despite benefiting firms.

Finally, while Hotelling competition preserves the paper's central message, it changes the interpretation. In the monopoly model, certification creates a direct price premium for certification relative to the off-consortium benchmark, and the effort comparison is governed by consumers' willingness to pay for green value. Under Hotelling competition, prices are driven by relative certification status, and the effort comparison is governed by the off-consortium benchmark effort. Overall, regardless of the mechanism, consortia can increase firm profit while weakening sustainability incentives and reducing consumer or social welfare as a result.

7 Conclusion

The emergence of eco-conscious consumers has driven firms to adopt sustainable production practices. To address the challenge of verifying sustainability claims, some of these firms, including luxury brands like Louis Vuitton and Mercedes-Benz, have formed decentralized consortia like Aura to certify each other's products. In this paper, we provide a theoretical investigation into the impact of decentralized consortia on the sustainability incentives of firms. Our analyses reveal that while joining a consortium allows firms to increase their profits by unlocking the 'green' premium consumers are willing to pay for certified products, it can paradoxically lead to lower firm sustainability efforts compared to a benchmark without certification. Specifically, when a firm is already exerting high sustainability effort, the certified price premium has limited room to grow further and cannot offset the risk of wasted effort if products are found to be non-green and fail to be certified, a situation that typically arises in markets with high consumer willingness to pay for green products. In this case, firms reduce their sustainability effort to mitigate the potential risk. For consumers, this reduction in the firms' sustainability efforts can undermine the actual value delivered by certification. Our findings indicate that for sufficiently high levels of consumers' willingness to pay for green products, decentralized consortia reduce consumer surplus and, despite improving firm profits, may ultimately lead to a net loss in social welfare. These findings remain robust when consortium members compete in the product market: even under Hotelling competition, the consortium can still reduce sustainability effort and consumer welfare while always increasing firm profits.

The paper provides strategic insights for both consortia executives and regulators. For executives, joining a consortium reliably raises firm profits across market types. However, careful consideration of consumer characteristics is crucial, because the consortium's impact on sustainability and social welfare diverges sharply by market. In markets where consumers are less sensitive to green attributes, such as fast fashion or commodity goods, consortia enhance sustainability incentives and raise social welfare. Conversely, in markets where consumers are more eco-conscious, such as luxury goods and organic foods, the consortium weakens sustainability incentives and may reduce social welfare. Although decentralized consortia such as Aura have formed and remain successful in luxury and high-end markets because membership raises firm profits, our findings suggest these may paradoxically be the markets where consumer and social welfare are most likely to suffer. This effect is not driven simply by firms lacking concern for sustainability: even firms with intrinsic sustainability preferences may reduce effort on the consortium when baseline effort is sufficiently high. Furthermore, our competition extension shows that this pattern persists when consortium members compete in the product market.

For regulators, our findings highlight that certification does not always benefit all stakeholders. While the consortium reliably raises firm profits, it can come at the expense of consumer and social welfare, especially in markets where consumers are highly eco-conscious. Policy makers should therefore scrutinize decentralized consortia most closely in high-end and luxury markets, where firm and social interests may diverge most sharply.

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Online Appendix

Green Incentives in Decentralized Consortia

A Proofs

Proof of Lemma 1. Consumers' utility for purchasing firm i's product is characterized by (1), given e and p decided in the previous stage. There are two cases to consider: (1) $e = 0$, and (2) $0 < e \leq 1$.

First, we consider $e = 0$. In this case, $\mathcal{U}(v) = v_l - p$ for all v, so $\mathcal{U}(v) \geq 0 \iff p \leq v_l$, which means all consumers will purchase firm i's product at price p iff $p \leq v_l$. Otherwise, no consumers will purchase. Thus, we obtain $\hat{v} = v_l$ if $p \leq v_l$, and $\hat{v} = v_h$ if $p > v_l$.

Second, we consider $0 < e \leq 1$. In this case, $\frac{\partial \mathcal{U}}{\partial v} = e > 0$, so $\mathcal{U}(v)$ is strictly increasing in v. Thus, $\mathcal{U}(v) \geq 0 \iff v \geq \frac{p-(1-e)v_l}{e}$. We obtain a unique threshold $\hat{v} = \frac{p-(1-e)v_l}{e}$ such that consumers with $v \geq \hat{v}$ will purchase firm i's product, and consumers with $v < \hat{v}$ will not. ■

Lemma A.1 *(Equilibrium Product Price Off-Consortium) In the off-consortium game, for a fixed sustainability effort e, firm i's equilibrium product price* p^* *is*

$$p^* = \begin{cases} v_l & \textit{if } 0 \leq e \leq \frac{v_l}{v_h - v_l}, \\ \frac{ev_h + (1-e)v_l}{2} & \textit{if } \frac{v_l}{v_h - v_l} < e \leq 1. \end{cases} \tag{23}$$

Proof of Lemma A.1. We use backward induction to solve the game. First, in stage 2 (the selling period), consumers' utility for purchasing firm i's product is characterized by (1), given e and p decided in the previous stage. There are two cases to consider: (1) $e = 0$, and (2) $0 < e \leq 1$.

First, we consider $e = 0$. In this case, $\mathcal{U}(v) \geq 0 \iff p \leq v_l$, which means all consumers will purchase Firm i's product at price p iff $p \leq v_l$. Otherwise, no consumers will purchase. In the former case, the demand for Firm i's product given e and p in stage 2 is:

$$D = \int_{v_l}^{v_h} \frac{1}{v_h - v_l} \cdot \mathbb{1}_{\mathcal{U}(v) \geq 0} \, dv = \int_{v_l}^{v_h} \frac{1}{v_h - v_l} \, dv = \frac{v_h - v_l}{v_h - v_l} = 1. \tag{24}$$

Next, in stage 1 (the production period), where Firm i chooses the price of the product p, to maximize $\Pi^0(p) = p \cdot D - t(e) = p - t(e)$, given e decided earlier in stage 1. Plugging (24) into the objective function in (3), Firm i's problem when choosing p in stage 1 becomes trivial:

$\max_{p \le v_l} p - t(e)$, which implies the optimal price is at the boundary

$$p = v_l = \bar{v} - \delta. \tag{25}$$

Second, we consider $0 < e \le 1$. In this case, $\mathcal{U}(v) \ge 0 \iff v \ge \frac{p-(1-e)v_l}{e}$, so the demand for Firm i's product given e and p in stage 2 is as follows:

$$D(e,p) = \begin{cases} 1, & p \le v_l, \\ \dfrac{v_l + 2\delta e - p}{2\delta e}, & v_l < p \le v_l + 2\delta e, \\ 0, & p > v_l + 2\delta e. \end{cases}$$

Next, in stage 1 (the production period), Firm i chooses p to maximize $\Pi^0(p) = p \cdot D - t(e)$. The unconstrained first-order condition on the interior branch ($v_l < p \le v_h$) yields

$$\frac{d\Pi^0(p)}{dp} = \frac{\delta(-1+2e) - 2p + \bar{v}}{2\delta e}, \qquad \frac{d^2\Pi^0(p)}{dp^2} = -\frac{1}{\delta e} < 0,$$

and the unconstrained interior maximizer is $\tilde{p} = \frac{\bar{v}+2\delta e-\delta}{2} = \frac{ev_h+(1-e)v_l}{2}$. We have $\tilde{p} \ge v_l$ if and only if $e \ge \frac{v_l}{v_h - v_l}$.

(i) If $0 < e \le \frac{v_l}{v_h-v_l}$, then $\tilde{p} < v_l$, the interior FOC is infeasible, and demand is at its upper-branch boundary $D = 1$. Firm i's problem reduces to $\max_{p \le v_l} p - t(e)$, with the corner solution

$$p = v_l = \bar{v} - \delta. \tag{26}$$

(ii) If $\frac{v_l}{v_h-v_l} < e \le 1$, then $\tilde{p} \ge v_l$, the interior branch is active, and the optimal price is

$$p = \frac{\bar{v} + 2\delta e - \delta}{2} = \frac{ev_h + (1-e)v_l}{2}. \tag{27}$$

To summarize, we obtain Firm i's equilibrium product price in stage 1, for given e chosen by Firm i earlier in the stage, in the off-consortium game as follows:

(1) If $0 \le e \le \frac{v_l}{v_h-v_l}$, by (25) and (26), we obtain the unique equilibrium product price $p^* = v_l$.

(2) If $\frac{v_l}{v_h-v_l} < e \le 1$, by (27), we obtain the unique equilibrium product price $p^* = \frac{ev_h+(1-e)v_l}{2}$. ■

Proof of Proposition 1. Based on the equilibrium product price in stage 1 derived in Lemma A.1, the demand for firm i's product for fixed sustainability effort e, $D(e)$, in the off-consortium game is

$$D(e) = \begin{cases} 1 & \text{if } 0 \le e \le \frac{v_l}{v_h - v_l}, \\ \frac{\delta(-1+2e)+\bar{v}}{4\delta e} & \text{if } \frac{v_l}{v_h - v_l} < e \le 1. \end{cases} \tag{28}$$

Next, consider the early period of stage 1 (the production period), where firm i chooses its sustainability effort e, to maximize $\Pi^0(e) = p \cdot D - t(e)$. Plugging (23) and (28) into the objective function in (3), we have firm i's problem in stage 1 as follows:

$$\max_{e} \quad p^* \cdot D(e) - t(e).$$

By Lemma A.1, there are two relevant effort regions. If $0 \le e \le v_l/(2\delta)$, then the optimal price is $p^*(e) = v_l$, demand is one, and profit is $\Pi_1^0(e) = v_l - t(e)$, which is strictly decreasing in e. Hence the maximum on this region is attained at $e = 0$. If $e > v_l/(2\delta)$, then

$$p^*(e) = \frac{v_l + 2\delta e}{2}$$

and

$$D(e, p^*(e)) = \frac{v_l + 2\delta e}{4\delta e}.$$

Thus

$$\Pi_2^0(e) = \frac{(v_l + 2\delta e)^2}{8\delta e} - t(e).$$

The first and second-order conditions with respect to e yield

$$\frac{d\Pi_2^0(e)}{de} = \frac{4\delta^2 e^2 - (-\delta + \bar{v})^2}{8\delta e^2} - t'(e), \qquad \frac{d^2\Pi_2^0(e)}{de^2} = \frac{(\bar{v} - \delta)^2}{4\delta e^3} - t''(e).$$

By the standing assumption on t (Section 3.1), the interior FOC admits a unique critical point e^* at which the second-order condition holds, i.e., $t''(e^*) > (\bar{v} - \delta)^2/(4\delta e^{*3})$, so $\frac{d^2\Pi_2^0}{de^2}\big|_{e^*} < 0$ and e^* is a local maximum of Π_2^0 on $(v_l/(v_h - v_l), 1]$. Hence, firm i's optimal sustainability effort on this region is uniquely determined by

$$t'(e^*) = \frac{4\,\delta^2\, e^{*2} - (\bar{v} - \delta)^2}{8\delta e^{*2}}$$

with associated optimal expected profit

$$\Pi_2^0(e^*) = \frac{(-\delta + 2\delta e^* + \bar{v})^2}{8\delta e^*} - t(e^*).$$

Next, we compare firm i's optimal expected profit in the two regions of e. By the standing assumption, the interior critical point strictly dominates the corner solution, i.e., $\Pi_2^0(e^*) > \Pi_1^0(0) = v_l - t(0)$. Therefore, firm i's equilibrium sustainability effort satisfies $e^* > v_l/(v_h - v_l)$ and is uniquely determined by $t'(e^*) = \frac{4\,\delta^2\,e^{*2} - (\bar{v}-\delta)^2}{8\delta e^{*2}}$.

By plugging firm i's equilibrium sustainability effort, e^*, into Lemma A.1, we can obtain firm i's equilibrium product price, p^*, in the off-consortium game.

Finally, we show that e^* and p^* are increasing in $\bar{v}$. Substituting $\delta = \bar{v} - v_l$ into (4) and implicitly differentiating with respect to $\bar{v}$, we obtain that the derivative of the RHS with respect to $\bar{v}$ is $\frac{4e^{*2}(\bar{v}-v_l)^2 + v_l^2}{8e^{*2}(\bar{v}-v_l)^2}$, which is strictly positive for all $e^* > 0$ and $\bar{v} > v_l$. Thus, e^* is increasing in $\bar{v}$. Since $p^* = \frac{e^* v_h + (1-e^*)v_l}{2}$ is increasing in both e^* and $v_h = 2\bar{v} - v_l$, we obtain that p^* is increasing in $\bar{v}$. ■

Proof of Lemma 2. In the selling period, consumers' utility for purchasing firm i's certified product is characterized by (7), given e and p_h decided in the previous stages. There are two cases to consider: (1) $e = 0$, and (2) $0 < e \leq 1$.

First, we consider $e = 0$. In this case, $\mathcal{U}_h(v) = v_l - p_h$, which means all consumers will purchase firm i's certified product at price p_h iff $p_h \leq v_l$. Otherwise, no consumers will purchase. Thus, we obtain $\hat{v} = v_l$ if $p_h \leq v_l$, and $\hat{v} = v_h$ if $p_h > v_l$.

Second, we consider $0 < e \leq 1$. In this case, $\frac{\partial \mathcal{U}_h}{\partial v} = \frac{e}{e+(1-e)(1-\alpha)} > 0$, so $\mathcal{U}_h(v)$ is strictly increasing in v. Thus, $\mathcal{U}_h(v) \geq 0 \iff v \geq \frac{p_h - \frac{(1-\alpha)(1-e)v_l}{e+(1-\alpha)(1-e)}}{\frac{e}{e+(1-\alpha)(1-e)}}$. We obtain a unique threshold $\hat{v}$ such that consumers with $v \geq \hat{v}$ will purchase firm i's certified product, and consumers with $v < \hat{v}$ will not. ■

Lemma A.2 *(Equilibrium On-Consortium Product Prices) In the on-consortium game, for a fixed sustainability effort e, firm i's equilibrium product price when it is certified, ${p_h}^*$, and when it is non-certified, ${p_l}^*$, are*

$$
{p_h}^* = \begin{cases} v_l & \text{if } 0 \leq e \leq \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}, \\ \frac{e v_h + (1-\alpha)(1-e)v_l}{2(1-\alpha+\alpha e)} & \text{if } \frac{(1-\alpha)v_l}{2\delta-\alpha v_l} < e \leq 1, \end{cases} \tag{29}
$$

$$
{p_l}^* = v_l.
$$

Proof of Lemma A.2. We use backward induction to solve the game. First, in stage 3 (the selling period), consumers' utility for purchasing firm i's product when it is certified and non-certified is characterized by (7) and (5), respectively, given e and p decided in the previous stages. There are two cases to consider: (1) $e = 0$, and (2) $0 < e \leq 1$.

First, we consider $e = 0$. In this case, $\mathcal{U}_l(v) \geq 0 \iff p_l \leq v_l$, which means all consumers will purchase firm i's non-certified product at price p_l iff $p_l \leq v_l$. Otherwise, no consumers will purchase. Similarly, $\mathcal{U}_h(v) \geq 0 \iff p_h \leq v_l$. Thus, in the former case, the demand for firm i's product when it is certified and non-certified are $D_h = 1$ and $D_l = 1$, respectively. Next, in stage 2 (the validation and pricing period), where firm i chooses its product price for when the product is certified, p_h, and for when the product is non-certified, p_l, to maximize $\Pi^{\mathcal{C}}(p_h, p_l) = (e + (1-e)(1-\alpha))p_h D_h + \alpha(1-e)p_l \cdot D_l - t(e)$, given e decided earlier in stage 1. Plugging D_h and D_l into the objective function in (9), we have firm i's problem when choosing p_h and p_l in stage 2 as follows:

$$\max_{p_h, p_l} \quad (1-\alpha) \cdot p_h + \alpha \cdot p_l - t(e).$$

Since $p_h \leq v_l$ and $p_l \leq v_l$, which implies the optimal prices are at the boundary

$$p_h{}^* = v_l = \bar{v} - \delta, \qquad p_l{}^* = v_l = \bar{v} - \delta. \tag{30}$$

Second, we consider $0 < e \leq 1$. In this case, on the interior branch where the consumer threshold satisfies $\hat{v} \in (v_l, v_h]$, $\hat{v} = \frac{p_h - \frac{(1-\alpha)(1-e)}{e+(1-\alpha)(1-e)} v_l}{\frac{e}{e+(1-\alpha)(1-e)}}$, and the demand for firm i's product when it is certified is $D_h = \frac{ev_h - (e+(1-\alpha)(1-e))p_h + (1-\alpha)(1-e)v_l}{2\delta e}$. The demand for firm i's product when it is non-certified is $D_l = 1$ if $p_l \leq v_l$ and $D_l = 0$ otherwise. Plugging D_h and D_l into the objective function in (9), the first-order condition for p_h on the interior branch yields

$$\frac{d\Pi^{\mathcal{C}}}{dp_h} = \frac{(1+\alpha(-1+e))\big(ev_h - 2(1-\alpha+\alpha e)p_h + (1-\alpha)(1-e)v_l\big)}{2\delta e}, \qquad \frac{d^2\Pi^{\mathcal{C}}}{dp_h^2} = -\frac{(1-\alpha+\alpha e)^2}{\delta e} < 0,$$

and the unconstrained interior maximizer is

$$\tilde{p}_h = \frac{ev_h + (1-\alpha)(1-e)v_l}{2(1-\alpha+\alpha e)}.$$

We have $\tilde{p}_h \geq v_l$ if and only if $e \geq \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}$.
(i) If $0 < e \leq \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}$, then $\tilde{p}_h < v_l$, the interior FOC is infeasible, and demand at $p_h = v_l$ is at the upper-branch boundary $D_h = 1$. Firm i's certified-state problem reduces to $\max_{p_h \leq v_l} p_h - t(e)$, with the corner solution

$$p_h^* = v_l = \bar{v} - \delta. \tag{31}$$

(ii) If $\frac{(1-\alpha)v_l}{2\delta - \alpha v_l} < e \leq 1$, the interior maximizer applies, and

$$p_h^* = \frac{ev_h + (1-\alpha)(1-e)v_l}{2(1-\alpha+\alpha e)}. \tag{32}$$

For the non-certified price, $\frac{d\Pi^{\mathcal{C}}}{dp_l} = \alpha(1-e) \geq 0$ on the interior $D_l = 1$ branch ($p_l \leq v_l$), so the firm sets p_l at the upper boundary, $p_l^* = v_l = \bar{v} - \delta$. ■

Proof of Proposition 2. Next, consider the first decision in stage 1 (the production period), where firm i chooses its sustainability effort e to maximize $\Pi^{\mathcal{C}} = (e + (1-e)(1-\alpha)) \cdot p_h \cdot D_h + \alpha(1-e) \cdot p_l \cdot D_l - t(e)$. By Lemma A.2, there are two different regions of e for firm i: (1) $0 \leq e \leq \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}$, and (2) $\frac{(1-\alpha)v_l}{2\delta - \alpha v_l} < e \leq 1$.

First, we consider $0 \leq e \leq \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}$. In this case, $p_h^* = v_l$ with $D_h = 1$, and $p_l^* = v_l$ with $D_l = 1$, so the objective is $\Pi^{\mathcal{C}}(e) = v_l - t(e)$, which is strictly decreasing in e (since $t'(e) > 0$). The maximum on this region is attained at the boundary $e = 0$, with profit $\Pi_1^{\mathcal{C}}(0) = v_l - t(0)$.

Second, we consider $\frac{(1-\alpha)v_l}{2\delta - \alpha v_l} < e \leq 1$. In this case, the first and second-order conditions of $\Pi^{\mathcal{C}}$ with respect to e yield $\frac{d\Pi^{\mathcal{C}}}{de} = \frac{e^2((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^2} - t'(e)$, $\frac{d^2\Pi^{\mathcal{C}}}{de^2} = \frac{(1-\alpha)^2(\bar{v}-\delta)^2}{4\delta e^3} - t''(e)$. By the standing assumption on t (Section 3.1), the interior FOC admits a unique critical point e^* at which the second-order condition holds, i.e., $t''(e^*) > (1-\alpha)^2(\bar{v}-\delta)^2/(4\delta e^{*3})$, so $\frac{d^2\Pi^{\mathcal{C}}}{de^2}\big|_{e^*} < 0$ and e^* is a local maximum of $\Pi^{\mathcal{C}}$ on $(\frac{(1-\alpha)v_l}{2\delta - \alpha v_l}, 1]$. Hence, firm i's optimal sustainability effort on this region is uniquely determined by

$$t'(e^*) = \frac{e^{*2}((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^{*2}}. \tag{33}$$

Next, we compare firm i's optimal expected profit in the two regions. By the standing assumption, the interior critical point strictly dominates the corner solution, i.e., $\Pi_2^{\mathcal{C}}(e^*) > \Pi_1^{\mathcal{C}}(0) = v_l - t(0)$. Therefore, firm i's equilibrium sustainability effort satisfies $e^* > \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}$ and is uniquely determined by (33). Finally, by plugging firm i's equilibrium sustainability effort, e^* into Lemma A.2, we can obtain firm i's equilibrium prices when its product is certified, ${p_h}^*$, and when its product is non-certified, ${p_l}^*$, in the on-consortium game.

Finally, we show that e^* and ${p_h}^*$ are increasing in $\bar{v}$. Substituting $\delta = \bar{v} - v_l$ into (33) and implicitly differentiating with respect to $\bar{v}$, we obtain that the sign of $\frac{de^*}{d\bar{v}}$ is determined by

$$\frac{(1-\alpha)^2 v_l^2 + e^{*2}(4\bar{v}^2 - 8\bar{v}v_l + (4-\alpha^2)v_l^2)}{8e^{*2}(\bar{v}-v_l)^2} > 0,$$

which is strictly positive for all $e^* > 0$, $\alpha \in (0,1)$, and $\bar{v} > v_l$. Thus, e^* is increasing in $\bar{v}$. Furthermore, $\frac{\partial {p_h}^*}{\partial \bar{v}} = \frac{e^*}{1-\alpha+\alpha e^*} > 0$. Since e^* is increasing in $\bar{v}$ and ${p_h}^*$ is increasing in e^*, we obtain that ${p_h}^*$ is increasing in $\bar{v}$. ■

Proof of Proposition 3. We implicitly differentiate (10) with respect to α and obtain $\frac{de^*}{d\alpha}$ as follows:

$$\frac{de^*}{d\alpha} = \frac{(\delta - \bar{v})\left[\delta(1-\alpha+(2+\alpha)e^{*2}) - \bar{v}(1-\alpha+\alpha e^{*2})\right]}{4\delta e^{*2}\left[t''(e^*) - \frac{(1-\alpha)^2(\bar{v}-\delta)^2}{4\delta e^{*3}}\right]}.$$

By the sufficient convexity of $t(e)$, the denominator is positive. Using $\delta - \bar{v} = -v_l$ and $(2+\alpha)\delta - \alpha\bar{v} = 2\delta - \alpha v_l$, the bracket in the numerator simplifies to

$$\delta(1-\alpha+(2+\alpha)e^{*2}) - \bar{v}(1-\alpha+\alpha e^{*2}) = e^{*2}(2\delta - \alpha v_l) - (1-\alpha)\, v_l.$$

Since $\delta - \bar{v} = -v_l < 0$ and the denominator is positive, the sign of $\frac{de^*}{d\alpha}$ coincides with the sign of $(1-\alpha)v_l - e^{*2}(2\delta - \alpha v_l)$. Defining

$$\hat{e}^{\alpha} \equiv \sqrt{\frac{(1-\alpha)\, v_l}{2\delta - \alpha v_l}},$$

which is well-defined since $2\delta - \alpha v_l > 0$ on the on-consortium interior pricing region, we obtain that $\frac{de^*}{d\alpha} > 0$ if $e^* < \hat{e}^{\alpha}$, and $\frac{de^*}{d\alpha} < 0$ if $e^* > \hat{e}^{\alpha}$. Finally, the effort threshold condition can be interpreted as a cutoff in $\bar{v}$. From Proposition 2, the on-consortium equilibrium effort e^* is strictly increasing in $\bar{v}$. In contrast, the threshold $\hat{e}$ is strictly decreasing in $\delta = \bar{v} - v_l$. Therefore, whenever the equation $e^*(\bar{v}) = \hat{e}_{\alpha}(\bar{v})$ has a solution in the maintained interior region, that solution is unique. It follows that the condition $e^* < \hat{e}_{\alpha}$ corresponds to values of $\bar{v}$ below the cutoff, while $e^* > \hat{e}_{\alpha}$ correspond to values of $\bar{v}$ above it. ■

Proof of Proposition 4.

(i) By Lemma A.2, the on-consortium certified price can be written as $p_h^*(e) = (q_H(e)v_h + (1 - q_H(e))v_l)/2$ on the interior pricing branch, where $q_H(e) = e/(1-\alpha+\alpha e)$ is the consumer's posterior probability that a certified product is genuinely green, and $q_H(e) \geq e$ for $e \in [0,1]$ (with strict inequality for $\alpha > 0$ and $e \in (0,1)$). Thus, for any common effort e,

$$p_h^*(e) - p^*(e) = \delta\,(q_H(e) - e) = \frac{\alpha\,\delta\, e(1-e)}{1-\alpha+\alpha e} \geq 0,$$

with equality only when both prices are at the corner v_l (i.e., $e \leq \frac{(1-\alpha)v_l}{2\delta - \alpha v_l}$, the on-consortium interior-pricing threshold). For $p^*(e) \geq p_l^* = v_l$: we have $p^*(e) > v_l$ on the off-consortium interior region $e > v_l/(v_h - v_l)$ by Lemma A.1, and $p^*(e) = v_l = p_l^*$ otherwise. Both inequalities therefore hold with $\geq$ for all $e \in (0,1)$ and become strict at any equilibrium effort, since both e^{*0} and $e^{*\mathcal{C}}$ lie in their respective interior pricing regions.

(ii) Next, we compare the equilibrium sustainability effort levels between the off-consortium and on-consortium scenario. Since $e^{*\mathcal{C}}$ and e^{*0} are both strictly positive by Proposition 1 and 2, to compare $e^{*\mathcal{C}}$ and e^{*0}, we need to know how $\Pi^{\mathcal{C}}(e)$ changes at $e = e^{*0}$. By plugging in e^{*0}, as characterized in (4), into $\frac{d\Pi^{\mathcal{C}}(e)}{de}$, we have

$$\begin{aligned}\frac{d\Pi^{\mathcal{C}}(e)}{de}\bigg|_{e=e^{*0}} &= \frac{e^{*0^2}((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^{*0^2}} - t'(e^{*0}) \\ &= \frac{e^{*0^2}((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^{*0^2}} - \frac{4\,\delta^2\,e^{*0^2} - (\bar{v}-\delta)^2}{8\,\delta\,e^{*0^2}}.\end{aligned}$$

Using $\bar{v} - \delta = v_l$ together with the identities $[(2+\alpha)\delta - \alpha\bar{v}]^2 - 4\delta^2 = -\alpha v_l(4\delta - \alpha v_l)$ and $1 - (1-\alpha)^2 = \alpha(2-\alpha)$, this expression simplifies to

$$\frac{d\Pi^{\mathcal{C}}(e)}{de}\bigg|_{e=e^{*0}} = \frac{\alpha\, v_l\left[(2-\alpha)\,v_l \;-\; (4\delta - \alpha v_l)\,e^{*0^2}\right]}{8\,\delta\,e^{*0^2}}.$$

Since $\alpha, v_l > 0$ and $4\delta - \alpha v_l > 0$ on the on-consortium interior pricing region (which requires $2\delta - \alpha v_l > 0$), the sign of $\frac{d\Pi^{\mathcal{C}}(e)}{de}\big|_{e=e^{*0}}$ coincides with the sign of $(2-\alpha)v_l - (4\delta - \alpha v_l)\,e^{*0^2}$. Defining the threshold

$$\hat{e} \equiv \sqrt{\frac{(2-\alpha)\,v_l}{4\delta - \alpha v_l}},$$

the existence of an interior off-consortium equilibrium $e^{*0} \in (v_l/(2\delta), 1]$ requires $v_l < 2\delta$, which in turn gives $(2-\alpha)v_l < 4\delta - \alpha v_l$ and thus $\hat{e} \in (0, 1)$. Moreover, by Proposition 2, $\Pi^{\mathcal{C}}$ admits a unique interior critical point $e^{*\mathcal{C}}$ at which the second-order condition holds, so $\Pi^{\mathcal{C}}$ is single-peaked on its interior pricing region. Therefore $\frac{d\Pi^{\mathcal{C}}(e)}{de}\big|_{e=e^{*0}} > 0$ if $e^{*0} < \hat{e}$, in which case e^{*0} lies to the left of the on-consortium maximizer, so $e^{*\mathcal{C}} > e^{*0}$. Conversely, if $e^{*0} > \hat{e}$, then $\frac{d\Pi^{\mathcal{C}}(e)}{de}\big|_{e=e^{*0}} < 0$, e^{*0} lies to the right of the maximizer, and $e^{*\mathcal{C}} < e^{*0}$. The effort threshold condition also maps into a cutoff in $\bar{v}$. From Proposition 1, the off-consortium equilibrium effort e^{0*} is strictly increasing in $\bar{v}$. The threshold $\hat{e}$ is strictly decreasing in $\delta = \bar{v} - v_l$. Hence, whenever the equation $e^{0*}(\bar{v}) = \hat{e}(\bar{v})$ has a solution in the maintained interior region, the solution is unique. Thus, $e^{0*} < \hat{e}$ corresponds to lower values of $\bar{v}$, and $e^{0*} > \hat{e}$ otherwise.

(iii) By Proposition 1, firm i's off-consortium expected profit for a given e is $\Pi^0(e)$. Similarly, by Proposition 2, firm i's on-consortium expected profit for a given e is $\Pi^{\mathcal{C}}(e)$. For e in the off-consortium interior pricing region, the difference is

$$\Pi^{\mathcal{C}}(e) - \Pi^0(e) = \frac{\alpha(1-e)\,v_l\,[4\delta e - (2 - \alpha + \alpha e)\,v_l]}{8\delta e}.$$

At $e = e^{*0}$, the off-consortium FOC gives $4\delta e^{*0} > 2v_l \geq (2 - \alpha + \alpha e^{*0})\, v_l$, so $\Pi^{\mathcal{C}}(e^{*0}) > \Pi^0(e^{*0}) = \Pi^{*0}$. Since in equilibrium $e = e^{*\mathcal{C}}$,

$$\Pi^{*\mathcal{C}} = \Pi^{\mathcal{C}}(e^{*\mathcal{C}}) \geq \Pi^{\mathcal{C}}(e^{*0}) > \Pi^{*0}.$$

Thus, $\Pi^{*\mathcal{C}} > \Pi^{*0}$.

■

Proof of Proposition 5.

(i) We first derive closed-form expressions for the equilibrium consumer surplus in both games. By Proposition 1, in the off-consortium game, the equilibrium price is $p^{0*} = (v_l + 2\delta\, e^{0*})/2$, and a consumer with green-valuation v purchases if and only if her utility $\mathcal{U}(v) = e^{0*} v + (1 - e^{0*})\, v_l - p^{0*}$ is non-negative, i.e., $v \geq \hat{v}^0$, where $\hat{v}^0 = \bar{v} - v_l/(2\, e^{0*})$. Since $\mathcal{U}(v)$ is linear in v with slope e^{0*} and vanishes at $\hat{v}^0$, and $\bar{v} + \delta - \hat{v}^0 = (v_l + 2\delta\, e^{0*})/(2\, e^{0*})$, we obtain

$$CS^{0*} = \int_{\hat{v}^0}^{\bar{v}+\delta} \frac{\mathcal{U}(v)}{2\delta}\, dv \;=\; \frac{e^{0*}}{4\delta} \left(\bar{v} + \delta - \hat{v}^0\right)^2 \;=\; \frac{(v_l + 2\delta\, e^{0*})^2}{16\delta\, e^{0*}}.$$

Similarly, by Proposition 2, in the on-consortium game, certification occurs with probability $\beta = e^{\mathcal{C}*} + (1 - e^{\mathcal{C}*})(1 - \alpha)$, inducing posterior $q_H = e^{\mathcal{C}*}/\beta$ that the product is genuinely green. The certified-state price ${p_h}^*$ satisfies $\beta\, {p_h}^* = ((1 - \alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})/2$, and the certified-state cutoff $\hat{v}_h$ satisfies $\bar{v} + \delta - \hat{v}_h = {p_h}^*/q_H$. By the same argument as above, the consumer surplus in the certified state is $CS_h = ({p_h}^*)^2/(4\delta\, q_H)$, so the contribution to total consumer surplus is

$$\beta\, CS_h \;=\; \frac{\beta^2\, ({p_h}^*)^2}{4\delta\, e^{\mathcal{C}*}} \;=\; \frac{((1 - \alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{16\delta\, e^{\mathcal{C}*}},$$

where we used $\beta\, q_H = e^{\mathcal{C}*}$. In the non-certified state, the consumer's posterior of green is zero, so ${p_l}^* = v_l$ extracts all surplus and the contribution is 0. Therefore,

$$CS^{\mathcal{C}*} \;=\; \beta\, CS_h \;=\; \frac{((1 - \alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{16\delta\, e^{\mathcal{C}*}}.$$

Comparing the two closed-form expressions, $CS^{\mathcal{C}*} < CS^{0*}$ if and only if

$$\frac{((1 - \alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{e^{\mathcal{C}*}} \;<\; \frac{(v_l + 2\delta\, e^{0*})^2}{e^{0*}},$$

which establishes the if-and-only-if condition.

It remains to show that $CS^{\mathcal{C}*} < CS^{0*}$ whenever $e^{\mathcal{C}*} < e^{0*}$. To this end, define

$$\mu(e) \;:=\; \frac{v_l + 2\delta\, e}{\sqrt{e}}, \qquad \nu(e) \;:=\; \frac{(1-\alpha)\, v_l + (\alpha v_l + 2\delta)\, e}{\sqrt{e}},$$

so that $CS^{0*} = \mu(e^{0*})^2/(16\delta)$ and $CS^{\mathcal{C}*} = \nu(e^{\mathcal{C}*})^2/(16\delta)$. Since both μ, ν are positive, $CS^{\mathcal{C}*} < CS^{0*}$ if and only if $\nu(e^{\mathcal{C}*}) < \mu(e^{0*})$. Direct computation gives

$$\mu(e) - \nu(e) \;=\; \frac{\alpha\, v_l\,(1-e)}{\sqrt{e}} \;>\; 0 \quad \text{for all } e \in (0,1),$$

$$\mu'(e) \;=\; \frac{2\delta\, e - v_l}{2\, e^{3/2}}, \qquad \nu'(e) \;=\; \frac{(\alpha v_l + 2\delta)\, e - (1-\alpha)\, v_l}{2\, e^{3/2}}.$$

Hence μ attains its unique global minimum at $\underline{e}_\mu \equiv v_l/(2\delta)$, and ν at $\underline{e}_\nu \equiv (1-\alpha)\, v_l/(\alpha v_l + 2\delta)$. Since $2\delta(1-\alpha) < \alpha v_l + 2\delta$, we have $\underline{e}_\nu < \underline{e}_\mu$.

The off-consortium first-order condition in Proposition 1(i) yields

$$t'(e^{0*}) \;=\; \frac{(v_l + 2\delta\, e^{0*})(2\delta\, e^{0*} - v_l)}{8\delta\, (e^{0*})^2},$$

using $\bar{v} - \delta = v_l$ and $4\delta^2 e^2 - v_l^2 = (v_l + 2\delta e)(2\delta e - v_l)$. Since $t'(e^{0*}) > 0$, we have $e^{0*} > v_l/(2\delta) = \underline{e}_\mu$, so μ is strictly increasing at e^{0*}. Furthermore, the existence of an interior $e^{0*} \in (0,1)$ requires $\underline{e}_\mu < 1$, i.e., $v_l < 2\delta$.

Similarly, the on-consortium first-order condition in Proposition 2(i) simplifies, using $(2+\alpha)\delta - \alpha\bar{v} = 2\delta - \alpha v_l$, to

$$t'(e^{\mathcal{C}*}) \;=\; \frac{(2\delta - \alpha v_l)^2\, (e^{\mathcal{C}*})^2 - (1-\alpha)^2\, v_l^2}{8\delta\, (e^{\mathcal{C}*})^2}.$$

Since $t'(e^{\mathcal{C}*}) > 0$ and $2\delta - \alpha v_l > 0$ (as $v_l < 2\delta$ and $\alpha < 1$), we have $e^{\mathcal{C}*} > (1-\alpha)\, v_l/(2\delta - \alpha v_l) > (1-\alpha)\, v_l/(\alpha v_l + 2\delta) = \underline{e}_\nu$, so ν is strictly increasing at $e^{\mathcal{C}*}$.

Now suppose $e^{\mathcal{C}*} < e^{0*}$. We consider two cases. First, if $e^{\mathcal{C}*} \geq \underline{e}_\mu$, then μ is non-decreasing on $[e^{\mathcal{C}*}, e^{0*}]$, so $\mu(e^{\mathcal{C}*}) \leq \mu(e^{0*})$. Combined with the pointwise inequality $\nu(e^{\mathcal{C}*}) < \mu(e^{\mathcal{C}*})$, we obtain

$$\nu(e^{\mathcal{C}*}) \;<\; \mu(e^{\mathcal{C}*}) \;\leq\; \mu(e^{0*}).$$

Second, if $e^{\mathcal{C}*} < \underline{e}_\mu$, then since $e^{\mathcal{C}*} \in (\underline{e}_\nu, \underline{e}_\mu)$ and ν is strictly increasing on $[\underline{e}_\nu, \infty)$, we have $\nu(e^{\mathcal{C}*}) < \nu(\underline{e}_\mu)$. Furthermore, since $\underline{e}_\mu < 1$, the pointwise inequality at $e = \underline{e}_\mu$ gives $\nu(\underline{e}_\mu) < \mu(\underline{e}_\mu)$, and since μ attains its global minimum at $\underline{e}_\mu$, we have $\mu(\underline{e}_\mu) \leq \mu(e^{0*})$. Chaining these inequalities yields

$$\nu(e^{\mathcal{C}*}) \; < \; \nu(\underline{e}_\mu) \; < \; \mu(\underline{e}_\mu) \; \leq \; \mu(e^{0*}).$$

Thus, in both cases, $\nu(e^{\mathcal{C}*}) < \mu(e^{0*})$, which yields $CS^{\mathcal{C}*} < CS^{0*}$.

(ii) By Propositions 1 and 2, the firm's equilibrium expected profits are

$$\begin{aligned}
\Pi^{0*} &= \frac{(v_l + 2\delta\, e^{0*})^2}{8\delta\, e^{0*}} - t(e^{0*}), \\
\Pi^{\mathcal{C}*} &= \frac{((1-\alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{8\delta\, e^{\mathcal{C}*}} + \alpha\,(1 - e^{\mathcal{C}*})\, v_l - t(e^{\mathcal{C}*}).
\end{aligned}$$

Combining with the consumer-surplus expressions from part (i), the equilibrium social welfare in each game is

$$\begin{aligned}
W^{0*} &= \Pi^{0*} + CS^{0*} = \frac{3\,(v_l + 2\delta\, e^{0*})^2}{16\delta\, e^{0*}} - t(e^{0*}), \\
W^{\mathcal{C}*} &= \Pi^{\mathcal{C}*} + CS^{\mathcal{C}*} = \frac{3\,((1-\alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{16\delta\, e^{\mathcal{C}*}} + \alpha\,(1 - e^{\mathcal{C}*})\, v_l - t(e^{\mathcal{C}*}).
\end{aligned}$$

By Proposition 4(iii), $\Pi^{\mathcal{C}*} > \Pi^{0*}$. Hence, whenever $CS^{\mathcal{C}*} \geq CS^{0*}$,

$$W^{\mathcal{C}*} \; = \; \Pi^{\mathcal{C}*} + CS^{\mathcal{C}*} \; > \; \Pi^{0*} + CS^{0*} \; = \; W^{0*}.$$

It remains to derive the if-and-only-if condition for the case $e^{\mathcal{C}*} < e^{0*}$. Subtracting the welfare expressions above, we have

$$W^{\mathcal{C}*} - W^{0*} = \frac{3\,((1-\alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{16\delta\, e^{\mathcal{C}*}} - \frac{3\,(v_l + 2\delta\, e^{0*})^2}{16\delta\, e^{0*}} + \alpha\,(1 - e^{\mathcal{C}*})\, v_l - \big(t(e^{\mathcal{C}*}) - t(e^{0*})\big).$$

Thus, $W^{\mathcal{C}*} < W^{0*}$ if and only if

$$\frac{3\,(v_l + 2\delta\, e^{0*})^2}{16\delta\, e^{0*}} - \frac{3\,((1-\alpha)\, v_l + (\alpha v_l + 2\delta)\, e^{\mathcal{C}*})^2}{16\delta\, e^{\mathcal{C}*}} \; > \; \alpha\,(1 - e^{\mathcal{C}*})\, v_l + t(e^{0*}) - t(e^{\mathcal{C}*}),$$

which establishes the if-and-only-if condition.

■

Proof of Proposition 6.

(i) With the sustainability penalty, firm i's effort FOC in the off-consortium game becomes

$$t'(e) = \frac{4\delta^2 e^2 - (\bar{v}-\delta)^2}{8\delta e^2} + \gamma.$$

Implicitly differentiating with respect to γ, we obtain

$$\frac{de^{0*}}{d\gamma} = \frac{1}{t''(e^{0*}) - \frac{(\bar{v}-\delta)^2}{4\delta e^{0*3}}}.$$

By the sufficient convexity of $t(e)$, the denominator is positive, so $\frac{de^{0*}}{d\gamma} > 0$. Similarly, the on-consortium effort FOC becomes

$$t'(e) = \frac{e^2((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^2} + \gamma.$$

Implicitly differentiating with respect to γ, we obtain

$$\frac{de^{\mathcal{C}*}}{d\gamma} = \frac{1}{t''(e^{\mathcal{C}*}) - \frac{(1-\alpha)^2(\bar{v}-\delta)^2}{4\delta e^{\mathcal{C}*3}}}.$$

By the sufficient convexity of $t(e)$, the denominator is positive, so $\frac{de^{\mathcal{C}*}}{d\gamma} > 0$. Thus, both e^{0*} and $e^{\mathcal{C}*}$ are strictly increasing in γ.

(ii) To compare $e^{\mathcal{C}*}$ and e^{0*}, we evaluate $\frac{d\Pi^{\mathcal{C}}}{de}$ at $e = e^{0*}$. By plugging in e^{0*} into $\frac{d\Pi^{\mathcal{C}}}{de}$ and substituting the off-consortium FOC, we obtain

$$\left.\frac{d\Pi^{\mathcal{C}}}{de}\right|_{e=e^{0*}} = \frac{e^{0*2}((2+\alpha)\delta - \alpha\bar{v})^2 - (1-\alpha)^2(\bar{v}-\delta)^2}{8\delta e^{0*2}} - \frac{4\delta^2 e^{0*2} - (\bar{v}-\delta)^2}{8\delta e^{0*2}},$$

where the γ terms in the on-consortium derivative and the off-consortium FOC cancel, so the expression is identical to that in the proof of Proposition 4(ii). Applying the same simplification yields

$$\left.\frac{d\Pi^{\mathcal{C}}}{de}\right|_{e=e^{0*}} = \frac{\alpha\, v_l\,[(2-\alpha)\, v_l - (4\delta - \alpha v_l)\, e^{0*2}]}{8\delta\, e^{0*2}},$$

which is positive if and only if $e^{0*2} < (2-\alpha)v_l/(4\delta - \alpha v_l) = \hat{e}^2$. Thus, $e^{\mathcal{C}*} > e^{0*}$ if $e^{0*} < \hat{e}$, and $e^{\mathcal{C}*} < e^{0*}$ if $e^{0*} > \hat{e}$.

(iii) The threshold $\hat{e}$ depends only on the model primitives (α, v_l, δ), not on γ. By part (i), e^{0^*} is strictly increasing in γ. Therefore, the parameter region in which $e^{0^*} > \hat{e}$ (where the consortium reduces sustainability effort) expands as γ rises.

■

Lemma A.3 *(Hotelling Price Subgame) For any pair of posterior beliefs (q_i, q_j), the simultaneous-move pricing game between the two firms admits a unique interior equilibrium. The equilibrium prices are*

$$p_i(q_i, q_j) = \tau + \frac{\delta}{3}(q_i - q_j), \qquad p_j(q_i, q_j) = \tau + \frac{\delta}{3}(q_j - q_i), \tag{34}$$

firm i's equilibrium demand is

$$D_i(q_i, q_j) = \frac{1}{2} + \frac{\delta(q_i - q_j)}{6\tau}, \tag{35}$$

and firm i's equilibrium operating profit is

$$r_i(q_i, q_j) = p_i(q_i, q_j) \cdot D_i(q_i, q_j) = \frac{\tau}{2} + \frac{\delta}{3}(q_i - q_j) + \frac{\delta^2}{18\tau}(q_i - q_j)^2. \tag{36}$$

Proof of Lemma A.3. For a given realization of posterior beliefs (q_i, q_j) and prices (p_i, p_j), the consumer located at x with green-valuation realization z is indifferent between firm i and firm j when

$$\underline{v} + v_l + q_i z - p_i - \tau x = \underline{v} + v_l + q_j z - p_j - \tau(1 - x).$$

Solving for the indifferent location gives

$$x^*(z) = \frac{1}{2} + \frac{(q_i - q_j)z - p_i + p_j}{2\tau}.$$

Under the covered-market, interior-location assumption (for which $\tau > 4\delta/3$ is sufficient), firm i's demand is the expectation of $x^*(z)$ over $z \sim U[0, 2\delta]$, which yields D_i in (14). Firm i chooses p_i to maximize $p_i D_i$. The first-order condition is

$$\frac{\partial(p_i D_i)}{\partial p_i} = \frac{1}{2} + \frac{\delta(q_i - q_j) - 2p_i + p_j}{2\tau} = 0,$$

or equivalently $2p_i - p_j = \tau + \delta(q_i - q_j)$. Similarly, firm j's first-order condition is $2p_j - p_i = \tau + \delta(q_j - q_i)$. Solving these two linear equations gives

$$p_i = \tau + \frac{\delta}{3}(q_i - q_j), \qquad p_j = \tau + \frac{\delta}{3}(q_j - q_i).$$

Substituting these equilibrium prices back into (14) gives

$$D_i = \frac{1}{2} + \frac{\delta(q_i - q_j)}{6\tau}.$$

Finally,

$$r_i(q_i, q_j) = p_i D_i = \left(\tau + \frac{\delta}{3}(q_i - q_j)\right)\left(\frac{1}{2} + \frac{\delta(q_i - q_j)}{6\tau}\right) = \frac{\tau}{2} + \frac{\delta}{3}(q_i - q_j) + \frac{\delta^2}{18\tau}(q_i - q_j)^2.$$

■

Proposition A.1 *(Off-Consortium Equilibrium under Hotelling Competition) In the off-consortium Hotelling game with quadratic effort cost $t(e) = e^2$, there exists a unique symmetric interior equilibrium such that:*

(i) the firm's sustainability effort is

$$e^{0*} = \frac{\delta}{6}; \tag{37}$$

(ii) the firm's product price is $p^{0*} = \tau$, with each firm serving half of the market;

(iii) each firm's equilibrium profit is

$$\Pi^{0*} = \frac{\tau}{2} - (e^{0*})^2 = \frac{\tau}{2} - \frac{\delta^2}{36}. \tag{38}$$

Proof of Proposition A.1. Off the consortium, consumers form rational expectations about each firm's sustainability effort, so $q_i = e_i$ and $q_j = e_j$. Substituting these posteriors into Lemma A.3, firm i's realized operating profit is

$$r_i(e_i, e_j) = \frac{\tau}{2} + \frac{\delta}{3}(e_i - e_j) + \frac{\delta^2}{18\tau}(e_i - e_j)^2,$$

and its total expected profit is

$$\Pi_i^0(e_i, e_j) = r_i(e_i, e_j) - e_i^2.$$

The first-order condition with respect to e_i is

$$\frac{\partial \Pi_i^0}{\partial e_i} = \frac{\delta}{3} + \frac{\delta^2}{9\tau}(e_i - e_j) - 2e_i = 0.$$

At the symmetric equilibrium $e_i = e_j = e^{0*}$, this reduces to $2e^{0*} = \delta/3$, yielding $e^{0*} = \delta/6$. The second-order condition $\partial^2\Pi_i^0/\partial e_i^2 = \delta^2/(9\tau) - 2 < 0$ holds whenever $\tau > \delta^2/18$, which is implied by the covered-market condition $\tau > 4\delta/3$.

At the symmetric effort e^{0*}, Lemma A.3 gives equilibrium price $p^{0*} = \tau + \delta(e^{0*} - e^{0*})/3 = \tau$ and demand $D_i = 1/2$, so each firm's operating profit is $p^{0*} \cdot D_i = \tau/2$. The equilibrium per-firm profit is therefore

$$\Pi^{0*} = \frac{\tau}{2} - (e^{0*})^2 = \frac{\tau}{2} - \frac{\delta^2}{36}.$$

■

Proposition A.2 *(On-Consortium Equilibrium under Hotelling Competition) In the on-consortium Hotelling game with quadratic effort cost $t(e) = e^2$, with $V(e)$ as defined in equation (18), there exists a unique symmetric interior equilibrium such that:*

(i) the firm's sustainability effort $e^{\mathcal{C}*}$ satisfies $e^{\mathcal{C}*} > 0$ and is uniquely determined by the first-order condition

$$2e^{\mathcal{C}*} = \frac{\delta}{3} + \frac{\delta^2}{18\tau}V'(e^{\mathcal{C}*}); \tag{39}$$

(ii) letting $q^* \equiv q_h(e^{\mathcal{C}*}) = e^{\mathcal{C}*}/s(e^{\mathcal{C}*})$, the firm's equilibrium state-contingent prices are

$$\begin{aligned}
(h,h): &\quad p_i^{hh*} = p_j^{hh*} = \tau,\\
(h,l): &\quad p_i^{hl*} = \tau + \tfrac{\delta q^*}{3}, \quad p_j^{hl*} = \tau - \tfrac{\delta q^*}{3},\\
(l,h): &\quad p_i^{lh*} = \tau - \tfrac{\delta q^*}{3}, \quad p_j^{lh*} = \tau + \tfrac{\delta q^*}{3},\\
(l,l): &\quad p_i^{ll*} = p_j^{ll*} = \tau;
\end{aligned}$$

(iii) each firm's equilibrium profit is

$$\Pi^{\mathcal{C}*} = \frac{\tau}{2} + \frac{\delta^2}{9\tau}V(e^{\mathcal{C}*}) - (e^{\mathcal{C}*})^2. \tag{40}$$

Proof of Proposition A.2. Let $Q_i \in \{q_h(e_i), 0\}$ denote the random posterior belief about firm i's product after certification, which equals $q_h(e_i) = e_i/s(e_i)$ with probability $s(e_i)$ and 0 with probability $\ell(e_i) = \alpha(1 - e_i)$. Then

$$\mathbb{E}[Q_i] = s(e_i) \cdot q_h(e_i) + \ell(e_i) \cdot 0 = e_i,$$

and the variance is

$$\mathrm{Var}(Q_i) = s(e_i) \cdot q_h(e_i)^2 - e_i^2 = \frac{e_i^2}{s(e_i)} - e_i^2 = \frac{\alpha(1-e_i)e_i^2}{1-\alpha+\alpha e_i} = V(e_i).$$

Since certification outcomes are independent across firms, $\mathbb{E}[(Q_i - Q_j)^2] = V(e_i) + V(e_j) + (e_i - e_j)^2$. Taking the expectation of the realized operating profit in Lemma A.3 over the four certification states gives firm i's expected operating profit:

$$R_i^{\mathcal{C}}(e_i, e_j) = \frac{\tau}{2} + \frac{\delta}{3}(e_i - e_j) + \frac{\delta^2}{18\tau}\left[V(e_i) + V(e_j) + (e_i - e_j)^2\right], \tag{41}$$

and its total expected profit is $\Pi_i^{\mathcal{C}}(e_i, e_j) = R_i^{\mathcal{C}}(e_i, e_j) - e_i^2$. The first-order condition with respect to e_i is

$$\frac{\partial \Pi_i^{\mathcal{C}}}{\partial e_i} = \frac{\delta}{3} + \frac{\delta^2}{18\tau}\left[V'(e_i) + 2(e_i - e_j)\right] - 2e_i = 0.$$

At the symmetric equilibrium $e_i = e_j = e^{\mathcal{C}*}$, this reduces to

$$2e^{\mathcal{C}*} = \frac{\delta}{3} + \frac{\delta^2}{18\tau}V'(e^{\mathcal{C}*}),$$

establishing part (i). The second-order condition is $\partial^2 \Pi_i^{\mathcal{C}}/\partial e_i^2 = (\delta^2/(18\tau))[V''(e_i) + 2] - 2$. Analogous to the base model, we focus on parameter values for which the symmetric FOC admits a unique solution $e^{\mathcal{C}*} \in (0, 1)$ at which the SOC holds, i.e., $\tau > \delta^2[V''(e^{\mathcal{C}*}) + 2]/36$.

For part (ii), at the symmetric effort $e^{\mathcal{C}*}$, the posterior in state (x, y) is $(q_x(e^{\mathcal{C}*}), q_y(e^{\mathcal{C}*}))$ with $q_h(e^{\mathcal{C}*}) = q^*$ and $q_l = 0$. Substituting into the equilibrium price formula in Lemma A.3 yields the state-contingent prices stated in part (ii).

For part (iii), evaluating (41) at the symmetric effort $e^{\mathcal{C}*}$ gives $R_i^{\mathcal{C}}(e^{\mathcal{C}*}, e^{\mathcal{C}*}) = \tau/2 + (\delta^2/9\tau)V(e^{\mathcal{C}*})$, so

$$\Pi^{\mathcal{C}*} = R_i^{\mathcal{C}}(e^{\mathcal{C}*}, e^{\mathcal{C}*}) - (e^{\mathcal{C}*})^2 = \frac{\tau}{2} + \frac{\delta^2}{9\tau}V(e^{\mathcal{C}*}) - (e^{\mathcal{C}*})^2.$$

■

Proof of Proposition 7.

(i) At the symmetric on-consortium equilibrium $e^{\mathcal{C}*}$, Proposition A.2(ii) gives the state-contingent prices. Since the rival's certification status is independent of firm i's, conditional on firm i being certified, the rival is certified with probability $s(e^{\mathcal{C}*})$ and non-certified with probability

$\ell(e^{\mathcal{C}*})$. Hence

$$\bar{p}_h^* = s(e^{\mathcal{C}*}) \cdot \tau + \ell(e^{\mathcal{C}*}) \cdot \left(\tau + \frac{\delta q^*}{3}\right) = \tau + \frac{\delta}{3}\ell(e^{\mathcal{C}*})q^* = \tau + \frac{\delta}{3} \cdot \frac{\alpha(1-e^{\mathcal{C}*})e^{\mathcal{C}*}}{1-\alpha+\alpha e^{\mathcal{C}*}} > \tau = p^{0*}$$

whenever $\alpha > 0$ and $e^{\mathcal{C}*} \in (0,1)$. Conditional on firm i being non-certified,

$$\bar{p}_l^* = s(e^{\mathcal{C}*}) \cdot \left(\tau - \frac{\delta q^*}{3}\right) + \ell(e^{\mathcal{C}*}) \cdot \tau = \tau - \frac{\delta}{3}s(e^{\mathcal{C}*})q^* = \tau - \frac{\delta e^{\mathcal{C}*}}{3} < \tau = p^{0*},$$

where we used $s(e^{\mathcal{C}*})q^* = e^{\mathcal{C}*}$. The unconditional expected price is

$$\mathbb{E}[p_i^{\mathcal{C}*}] = s(e^{\mathcal{C}*})\bar{p}_h^* + \ell(e^{\mathcal{C}*})\bar{p}_l^* = \tau = p^{0*}.$$

(ii) The off-consortium FOC (Proposition A.1) is $2e^{0*} = \delta/3$ and the on-consortium FOC (Proposition A.2) is $2e^{\mathcal{C}*} = \delta/3 + (\delta^2/18\tau)V'(e^{\mathcal{C}*})$. Evaluating the on-consortium derivative at the off-consortium symmetric effort:

$$\left.\frac{\partial \Pi_i^{\mathcal{C}}}{\partial e_i}\right|_{e_i=e_j=e^{0*}} = \frac{\delta^2}{18\tau}V'(e^{0*}),$$

using the off-consortium FOC. By concavity, the sign of this derivative determines whether $e^{\mathcal{C}*}$ lies above or below e^{0*}.

Differentiating $V(e)$ gives

$$V'(e) = \frac{\alpha e\left[2(1-\alpha) + (4\alpha-3)e - 2\alpha e^2\right]}{(1-\alpha+\alpha e)^2}.$$

For $e \in (0,1)$, the prefactor is positive, so the sign of $V'(e)$ matches the sign of the bracketed term. The unique root of $2\alpha e^2 - (4\alpha-3)e - 2(1-\alpha) = 0$ in $(0,1)$ is

$$\widehat{e}(\alpha) = \frac{4\alpha - 3 + \sqrt{9-8\alpha}}{4\alpha},$$

with $\widehat{e}(1) = 1/2$ and $\lim_{\alpha\downarrow 0}\widehat{e}(\alpha) = 2/3$, so $\widehat{e}(\alpha) \in [1/2, 2/3]$. The bracketed term is positive for $e < \widehat{e}(\alpha)$ and negative for $e > \widehat{e}(\alpha)$. Therefore $e^{\mathcal{C}*} > e^{0*}$ if $e^{0*} < \widehat{e}(\alpha)$, and $e^{\mathcal{C}*} < e^{0*}$ if $e^{0*} > \widehat{e}(\alpha)$.

(iii) Let $K_\tau \equiv \delta^2/(18\tau)$. By Propositions A.1 and A.2,

$$\Pi^{0*} = \frac{\tau}{2} - (e^{0*})^2, \qquad \Pi^{\mathcal{C}*} = \frac{\tau}{2} + 2K_\tau V(e^{\mathcal{C}*}) - (e^{\mathcal{C}*})^2,$$

so

$$\Pi^{\mathcal{C}*} - \Pi^{0*} = 2K_\tau V(e^{\mathcal{C}*}) - \left[(e^{\mathcal{C}*})^2 - (e^{0*})^2\right].$$

Subtracting the off-consortium FOC from the on-consortium FOC gives $2(e^{\mathcal{C}*} - e^{0*}) = K_\tau V'(e^{\mathcal{C}*})$, so $e^{0*} = e^{\mathcal{C}*} - (K_\tau/2)V'(e^{\mathcal{C}*})$. Substituting,

$$(e^{\mathcal{C}*})^2 - (e^{0*})^2 = K_\tau e^{\mathcal{C}*} V'(e^{\mathcal{C}*}) - \frac{K_\tau^2}{4}\left[V'(e^{\mathcal{C}*})\right]^2.$$

Therefore

$$\Pi^{\mathcal{C}*} - \Pi^{0*} = K_\tau\left[2V(e^{\mathcal{C}*}) - e^{\mathcal{C}*}V'(e^{\mathcal{C}*})\right] + \frac{K_\tau^2}{4}\left[V'(e^{\mathcal{C}*})\right]^2.$$

Direct computation gives

$$2V(e) - eV'(e) = \frac{\alpha e^3}{(1-\alpha+\alpha e)^2} \geq 0$$

for all $e \in [0,1]$ and $\alpha \in (0,1]$, with strict inequality whenever $\alpha > 0$ and $e > 0$. Hence

$$\Pi^{\mathcal{C}*} - \Pi^{0*} = K_\tau \cdot \frac{\alpha(e^{\mathcal{C}*})^3}{(1-\alpha+\alpha e^{\mathcal{C}*})^2} + \frac{K_\tau^2}{4}\left[V'(e^{\mathcal{C}*})\right]^2 \geq 0,$$

with strict inequality whenever $\alpha > 0$ and $e^{\mathcal{C}*} > 0$.

■

Proof of Proposition 8.

(i) For realized posteriors (q_i, q_j) and equilibrium prices from Lemma A.3, the consumer at location x with green-valuation realization z is indifferent between firms at $x^*(z) = 1/2 + \Delta(z)/(2\tau)$, where $\Delta(z) \equiv (q_i - q_j)z - p_i + p_j$. Conditional on z, aggregate consumer surplus is

$$\begin{aligned} CS(z) &= \int_0^{x^*(z)} \mathcal{U}_i(x,z)\,dx + \int_{x^*(z)}^1 \mathcal{U}_j(x,z)\,dx \\ &= \underline{v} + v_l - \frac{p_i+p_j}{2} + \frac{q_i+q_j}{2}z - \frac{\tau}{4} + \frac{\Delta(z)^2}{4\tau}. \end{aligned}$$

In the pricing equilibrium, $p_i + p_j = 2\tau$ and $p_i - p_j = (2\delta/3)(q_i - q_j)$, so $\Delta(z) = (q_i - q_j)(z -$

$2\delta/3)$. Taking expectation over $z \sim U[0, 2\delta]$ and using $\mathbb{E}[z] = \delta$ and $\mathbb{E}[(z - 2\delta/3)^2] = 4\delta^2/9$ gives

$$CS(q_i, q_j) = \underline{v} + v_l + \frac{\delta}{2}(q_i + q_j) - \frac{5\tau}{4} + \frac{\delta^2}{9\tau}(q_i - q_j)^2.$$

Off the consortium, $q_i = q_j = e^{0*}$ at the symmetric equilibrium, so

$$CS^{0*} = \underline{v} + v_l + \delta e^{0*} - \frac{5\tau}{4}.$$

On the consortium, the random posteriors Q_i, Q_j at the symmetric equilibrium effort $e^{\mathcal{C}*}$ satisfy $\mathbb{E}[Q_i] = \mathbb{E}[Q_j] = e^{\mathcal{C}*}$ and, by independence, $\mathbb{E}[(Q_i - Q_j)^2] = 2V(e^{\mathcal{C}*})$. Hence

$$CS^{\mathcal{C}*} = \underline{v} + v_l + \delta e^{\mathcal{C}*} - \frac{5\tau}{4} + \frac{2\delta^2}{9\tau}V(e^{\mathcal{C}*}).$$

Therefore

$$CS^{\mathcal{C}*} - CS^{0*} = \delta(e^{\mathcal{C}*} - e^{0*}) + \frac{2\delta^2}{9\tau}V(e^{\mathcal{C}*}).$$

Since $V(e^{\mathcal{C}*}) \geq 0$, this is non-negative whenever $e^{\mathcal{C}*} \geq e^{0*}$, establishing $CS^{\mathcal{C}*} \geq CS^{0*}$ in that case. When $e^{\mathcal{C}*} < e^{0*}$, $CS^{\mathcal{C}*} < CS^{0*}$ if and only if $\delta(e^{0*} - e^{\mathcal{C}*}) > (2\delta^2/(9\tau))V(e^{\mathcal{C}*})$.

(ii) Social welfare is $W = CS + 2\Pi$. By Proposition A.1(iii), $\Pi^{0*} = \tau/2 - (e^{0*})^2$, and by Proposition A.2(iii), $\Pi^{\mathcal{C}*} = \tau/2 + (\delta^2/(9\tau))V(e^{\mathcal{C}*}) - (e^{\mathcal{C}*})^2$. Combining with the CS expressions above,

$$W^{0*} = \underline{v} + v_l + \delta e^{0*} - \frac{\tau}{4} - 2(e^{0*})^2,$$
$$W^{\mathcal{C}*} = \underline{v} + v_l + \delta e^{\mathcal{C}*} - \frac{\tau}{4} + \frac{4\delta^2}{9\tau}V(e^{\mathcal{C}*}) - 2(e^{\mathcal{C}*})^2,$$

and therefore

$$W^{\mathcal{C}*} - W^{0*} = \delta(e^{\mathcal{C}*} - e^{0*}) + \frac{4\delta^2}{9\tau}V(e^{\mathcal{C}*}) - 2\left[(e^{\mathcal{C}*})^2 - (e^{0*})^2\right].$$

When $e^{\mathcal{C}*} \geq e^{0*}$, part (i) gives $CS^{\mathcal{C}*} \geq CS^{0*}$ and Proposition 7(iii) gives $\Pi^{\mathcal{C}*} \geq \Pi^{0*}$, so $W^{\mathcal{C}*} \geq W^{0*}$. When $e^{\mathcal{C}*} < e^{0*}$, $W^{\mathcal{C}*} < W^{0*}$ if and only if $\delta(e^{0*} - e^{\mathcal{C}*}) > (4\delta^2/(9\tau))V(e^{\mathcal{C}*}) + 2[(e^{0*})^2 - (e^{\mathcal{C}*})^2]$.

■