The Enigma of Ticket Exchanges (and Other Reselling Markets)

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Abstract

The literature establishes (and practice confirms) that sellers can benefit from allowing consumers to purchase in advance of the date of actual consumption (e.g., concert tickets, sporting events, etc.). Because of this advance purchasing, consumers can find themselves either with a ticket that they no longer want, or without a ticket that they wish to have. In the past, scalpers would facilitate transactions among these consumers, for a fee. Sellers historically disliked those practices and actively worked to prevent them. In fact, we obtain a stark finding: an unfettered and efficient reselling market eliminates all of the benefits of advance selling, which justifies sellers' historic hostility to reselling. But now ticket exchanges are common, growing, and even embraced by the sellers. What changed? We present a theory that demonstrates reselling is actually beneficial for sellers under one crucial condition - the seller must have some control over the reselling process, thereby allowing the seller to earn something from each transaction through licensing fees to third-party sellers. The old-fashioned paper ticket did not give such control, but technology now enables electronic tickets, which do. In fact, a seller cannot earn more than what it receives from a properly designed reselling market (i.e., reselling is optimal for the seller), especially for popular events with limited capacity. Furthermore, speculators do not disrupt the market: i.e., the seller has no need for scalpers nor should fear them. In sum, our results explain why the seller's view towards reselling has shifted dramatically.

1 Introduction

Ticket exchanges, and other reselling markets, present an enigma. Not too long ago, sellers of tickets loathed reselling. And while consumers would purchase from scalpers, even consumers generally believed reselling to be problematic, risky, or unethical. Despite these stigmas, reselling markets have recently thrived. In fact, sellers of tickets now sanction such markets. Are they doing so begrudgingly, as in "if you can't beat them, join them"? Or is their support of these markets actually genuine, as in they recognize that these markets make them better off? And if so, why the change of heart? This paper develops a theory that answers these questions. In particular, we demonstrate that if a seller does not have control over the reselling market, and such a market exists, then not only does that market harm the seller, it also eliminates all of the well established advantages of selling in advance. Given such a result, it is clear why sellers would actively work to disrupt any reselling. However, if the seller can use technology to control the reselling market, as in earn

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a profit off of each transaction, then the conclusion radically changes. Now, not only does the presence of a reselling market enable the advantages of selling in advance, there does not exist a better way for the seller to sell, i.e., reselling is optimal (over all possible selling mechanisms). In short, enabled by technology that allows the tracking of tickets, a seller is able to regulate how reselling occurs, and by doing so the seller can (and should) now embrace reselling. Furthermore, such a reselling market is not hampered by the presence of speculators (those who purchase with no intention to consume).

Existing theory struggles to explain reselling. It has been well established in the literature that sellers can benefit through "advance selling" in which consumers are offered the opportunity to purchase the seller's service (e.g., tickets) well in advance of consumption, even before they are certain of the value they will receive from the service (e.g., Gale & Holmes 1993, Xie & Shugan 2001). Doing so allows the seller to sell more of its inventory than the seller would if the service were offered via "spot selling", which only allows consumers to purchase shortly before the service is offered when there is no longer any uncertainty regarding their preferences. Although advance selling can improve the seller's earnings, it comes with a cost. Some consumers who purchase in advance later learn they are holding a unit that they do not value much, and other consumers who were unable to purchase in advance find themselves without a unit that they strongly desire. A welfare enhancing exchange could occur between those consumers. But the analyses of advance selling explicitly or implicitly prohibit reselling between consumers, either on the presumption that it is not allowed (e.g., legal restrictions), or not feasible (e.g., too costly), or the seller can somehow eliminate it (and wants to eliminate it). In practice, especially for tickets to entertainment events, reselling has long occurred. In fact, some people who had no interest in the service would purchase in advance with the goal to resell to consumers at the time of the event ("on the spot"). Because they knew they had no actual value for the service, they were labeled "speculators", or in a more stigmatizing way, "scalpers". Given actual practice, ignoring the possibility of reselling from the theory of advance selling appears to be an important omission. In fact, we demonstrate that if the seller cannot prevent the operation of an efficient reselling market, then that market destroys the advantages of advance selling, as in the seller is unable to earn any more than what it would earn if the seller were to merely do spot selling. This is a stark result. It occurs because if consumers can resell, then they become future competitors to the seller, and a seller naturally wants to avoid competition. Hence, common sense suggests that sellers would oppose reselling - for advance selling to be beneficial, the seller needs to ensure that it does not face the competition created by an efficient reselling market.

If our model explains why a seller would oppose reselling (so that the seller can take advantage of advance selling), then why are ticket exchanges thriving (e.g., StubHub, etc.)? Furthermore, why are sellers supporting these reselling markets, through explicit agreements, rather than actively working to prevent them? For example, the National Football League (Fischer & Mccormick 2022), National Hockey League (Fischer 2013), and National Basketball Association (Fischer & Lombardo 2018) in the United States all have agreements with Ticketmaster, a leading resellers, to provide reselling services, and SeatGeek recently signed a $100 million deal to be the official reseller for Major League Baseball (Young, 2023). The answer comes from the theory of optimal mechanism design. We evaluate the best way for a seller to sell their services to consumers and discover that the optimal way to sell actually involves reselling. Although reselling can create competition for the seller, it can also increase the total value in the system, which can work to the seller's advantage. To explain, selling in advance comes with the cost of allocation errors - some consumers with a relatively low valuation for the service consume it nevertheless, while other consumers that substantially value the service are left wanting. Hence, there are welfare improving trades that could occur. If the seller is able to earn some of the value from those trades, then the seller can be even better off than merely selling in advance. But to do so requires that the seller have some control over the exchange to prevent excessive competition with resellers and to earn a portion of the trade's value. That control can be obtained via technology that allows the seller to monitor the exchange of ownership between consumers, i.e., consumers can sell their unit to other consumers but only through sanctioned electronic exchanges, i.e. an authorized reselling market. Once that technology becomes feasible, not only should the seller sell in advance, the seller should stop working against reselling and actually embrace it because there is no better way for the seller to earn revenue. Furthermore, authorized reselling eliminates the seller's concern with speculators - with a properly designed market there are no profitable opportunities for speculators and so they have no harmful (or beneficial) influence.

2 Related Literature

There is a large literature focused on selling capacity over time. An important body of this literature focuses on why selling in advance is advantageous, but that work tends to preclude (or ignore) the possibility of reselling. When reselling is explicitly considered, it is either taken as given that reselling is not desirable for the seller, or at least potentially harmful to the seller. Effort is made to identify conditions under which reselling could be beneficial for the seller. However, these conditions are somewhat restrictive or specialized (e.g., the seller has limited capacity, or the seller is required to charge a single price, etc.), whereas we find that authorized reselling can be broadly beneficial for the seller. Furthermore, studies of reselling do not claim to identify the seller's optimal mechanism, and more importantly, technology is viewed as only a means to prevent reselling rather than a tool for the seller to regulate and profit from reselling. The remainder of this section provides additional details on how this work is related to the literature.

Especially when selling perishable capacity well in advance of its usage, it is possible that consumers are initially somewhat uncertain regarding their preferences. Despite this uncertainty, for the right price, consumers may be willing to purchase in "advance", before they are certain of their value for the good, rather than "on the spot" when their preference uncertainty is resolved. An advance selling strategy, despite this uncertainty, can be highly effective for the seller: Gale & Holmes (1993) show that advance selling allows a monopolist seller to price discriminate between consumers who are relatively indifferent across products (e.g., peak and off-peak flights) and those that have stronger preferences; Dana (1998) shows that advance purchase discounts can arise in a competitive market; DeGraba (1995) demonstrates that a seller can be better off selling a limited amount of capacity in advance to consumers unsure of their preferences; Xie & Shugan (2001) emphasize that advance selling can be effective even with ample capacity; Chu & Zhang (2011) find that it is always in the seller's interest to sell to consumers with less than perfect preference information; advance selling can be used to update a seller's demand forecast (Moe & Fader 2002; Chu & Zhang 2011; Li & Zhang 2013); Cachon & Feldman (2011) show that advance selling via subscriptions can be effective even in services prone to congestion, despite the limited ability of subscriptions to control congestion and Nasiry & Popescu (2012) study advance selling with consumers that experience regret. Nevertheless, some limitations of advance selling have been identified: Xie & Shugan (2001) and Prasad *et al.* (2011) show that advance selling is not optimal if marginal costs are high, Cachon & Feldman (2017) show that advance selling can harm sellers by increasing the intensity of competition and Glazer *et al.* (2023) finds advance selling can be harmful when consumers prefer to purchase later but also fear rationing. Here, we find that advance selling provides no benefit to the seller over spot selling when capacity is scarce relative to demand. None of these studies on advance selling consider reselling as a feasible option.

Selling durable goods creates the possibility of a market for used goods, which resembles reselling. The seller's primary challenge is that the current sales of product creates future competition when those units become available as used goods (Levinthal & Purohit 1989; Purohit & Staelin 1994; Chen *et al.* 2013). Several strategies for the seller have been identified to mitigate this problem: the seller can lease its product instead of outright selling it (e.g. Desai & Purohit 1998); the seller could buy back old items (Levinthal & Purohit 1989; Rao *et al.* 2009); or the seller can actively engage in the used good market (Shulman & Coughlan 2007). With these models consumers do not have uncertainty regarding their valuation of the good before they purchase, and in our setting the seller's good does not depreciate in quality over time, which means there is only a single good available rather than multiple goods that vary in quality (i.e., new and used products).

Early work on reselling focuses on individuals who do not value the seller's good, i.e., speculators. These resellers have been generally viewed as undesirable for a seller: Roth (2007) describes reselling by speculators as a repugnant transaction. For example, when late arriving consumers have higher valuations than early consumers, a seller might want to sell with an increasing price path. But Courty (2003) argues that speculators prevent the seller from implementing that strategy because they create competition to sell to the high value consumers. However, consumer reselling is not considered.

Several papers document the problem of underpricing (i.e., the seller's tendency to set a price at a level at which demand exceeds supply), which may arise due to the difficulty in pricing hard-to-price goods or the desire to price fairly. Regardless of the reason, pricing below market invites speculation and reduces revenue. Instead of banning resale, which is practically difficult, these papers offer solutions to circumvent entry by speculators and increase revenue. Bhave & Budish (2017) propose running auctions to allow better price discovery and Courty (2019) proposes a full refund for returned units which are then randomly allocated. In contrast to our model, both papers assume that customers are aware of their value for the unit and that the seller does not or cannot set the optimal price in the primary market.

A number of studies seek to justify why reselling can benefit a seller. Geng *et al.* (2007) finds that a seller under certain demand and capacity conditions may benefit from reselling between consumers in an advance period as long as the seller is able to prevent reselling in the spot period, because reselling in the spot period would eliminate the scarcity risk that motivates consumers to pay more for the item in the advance period. However, with current reselling markets exchanges are feasible right up to the moment of the event (Cui *et al.* 2014). Su (2010) and Cui *et al.* (2014) find that speculators may indirectly allow a seller to implement dynamic pricing when the seller is unable or unwilling to adjust prices over time. We confirm this result in our model. If the seller cannot charge different prices across time, and if early demand is insufficient to sell all of the seller's capacity, then the seller can benefit from speculators purchasing in advance and selling later on the spot period for a higher price because consumers are willing to pay more in advance to avoid the higher prices caused by the speculators on the spot. However, speculators no longer are useful for the seller if the seller has flexibility with its pricing or if early demand is ample. Kuksov & Liao (2023) also finds that speculators can be beneficial to the firm because their presence raises the spot price (speculators only want to "buy low and sell high"), which motivates regular consumers to pay more in the initial period to avoid the higher prices in the spot period. But the seller must be able to limit the number of sales to speculators (if they buy too much, the competition they create actually reduces the spot period price), and they find that reselling between consumers is never in the seller's interest. Thus, Kuksov & Liao (2023) suggest that the seller benefits from a reselling channel that is welcoming to speculators but also excludes consumers. Karp & Perloff (2005) argue that speculators are better able to price better than other consumers or the seller. In particular, they can set a price that matches each seller's maximum willingness to pay. It is not clear why speculators would have this advantage over the seller in many situations, but if they do have this extra ability, then the seller can gain from trading with these speculators. Swofford (1999) suggests that speculators have less risk aversion than consumers, so the insurance they can provide in the market is useful to the seller. In our model all agents are equally skilled at pricing and all are risk neutral.

We presume consumers have either very high transaction costs (and hence they are unable to resell) or very low transaction costs (the reselling market is efficient), whereas Cui *et al.* (2014) consider intermediate levels of transaction costs. The availability of inexpensive information technology has likely reduced these frictions, thereby enabling efficient consumer-to-consumer reselling exchanges (e.g., StubHub). Nevertheless, as we discuss in Section 5, intermediate levels do not conceptually alter our results. As far as we are aware, we are the only study that allows the seller to earn transaction fees on trades that occur in the reselling market. The seller could charge these transaction fees directly via ownership of the reselling market or, more likely, through careful contractual control of a third-party. See Zou & Jiang (2020) for a discussion of the impact of an independent reselling market.

As in our model in which consumers arrive sequentially, Yang *et al.* (2017) consider reselling positions in a queue. However, consumers in their model do not learn information over time regarding their valuation and they do not consider dynamic pricing. Nevertheless, in their setting they demonstrate that social welfare and seller profits can increase substantially by allowing consumers to resell.

In sum, previous work has identified some situations in which reselling can work to a seller's advantage, but these cases generally involve specialized circumstances, such as exogenously imposed restrictions on the seller's ability to change prices, or established channels for speculators that exclude consumers from reselling, or capabilities speculators have (as in perfect price discrimination) that the seller lacks. As such, reselling has yet to be identified as effective for the seller in broad circumstances in which there are low market frictions and agents all act rationally, nor has reselling been identified as part of the seller's optimal mechanism. The primary problem with reselling is that it creates competition between the seller and whomever is reselling. However, reselling also generates value by allowing consumers who do not really want their unit to sell their unit to consumers that really do. Crucially, the seller needs technology not to prevent reselling, but to allow the seller to control and profit from reselling. This control allows the seller to ensure that reselling does not create destructive competition while also ensuring that the seller's capacity is best allocated to the consumers who value it the most, and therefore are willing to pay the most.

3 Model

A seller sells a service that is used at a particular point in time, such as admission to an entertainment event, transportation, or lodging. The seller has a fixed and limited amount of capacity. Consumers can anticipate in advance of the service event that they may value the service, but their precise preferences are only revealed closer to the time of the event. The seller can sell its capacity over time, e.g. well in advance, or closer to "on the spot" just before when the capacity is used, among many feasible selling mechanisms. Speculators have the opportunity to purchase in advance with the goal to sell later for a profit.

The interactions among the consumers, speculators and the seller occur over a two period time horizon. The seller has the capacity to serve q units of demand. The seller's good is provided at the end of the horizon. Period 1 is referred to as the "advance" period and period 2 is the "spot" period. The seller incurs zero marginal cost to deliver its product and unused capacity at the end of the horizon is wasted because the seller receives no value for unsold capacity. (Equivalently, the marginal cost and salvage value are normalized to zero.)

The market consists of a mass of $n = n_1 + n_2$ consumers who seek the seller's good, where n_i is the mass of consumers who arrive in period i. There is potentially more demand than capacity, i.e. $q < n$. Hence, scarcity and rationing may occur with this product. Each consumer is sufficiently small in the market, so the number of consumers (n, n_1, n_2) and the available capacity (q) can be taken to be continuous variables. For notational convenience, let κ be the ratio of capacity to demand, $\kappa = q/n$, and let λ_1 be the ratio of period 1 demand to total demand, $\lambda_1 = n_1/n$. Equivalently, κ and λ_1 are capacity and period 1 demand per unit of total demand, or capacity and period 1 demand after normalizing total demand to one. All results depends on these ratios of capacity and period 1 demand to total demand (κ and λ_1) rather than their absolute levels (q and n_1).

At the start of period 1 each consumer only knows their value for the good is uniformly distributed on the $[0, 1]$ interval. They also know everyone observes their own value at the start of period 2 and values are identically distributed and independent across consumers. For example, a consumer may know (in advance) that she will have some value for celebrating a daughter's birthday at a basketball game, but she also knows that she only learns closer to the time of the event exactly how much she values it. See Papanastasiou & Savva (2017) and Feldman *et al.* (2019) for models in which consumer learning is endogenously determined by the seller's actions rather than, as in our model, an exogenous process.

Besides regular consumers, there is an unlimited pool of speculators present in period 1. Speculators receive no value from the seller's product. So speculators purchase in period 1 only if they believe they can earn a profit by selling the good in period 2. Speculators are equivalent to consumers with zero value, but given their large mass, it is useful to given them this special classification.

The core design of our model is similar to others that have been considered in the literature (Geng *et al.* 2007; Cui *et al.* 2014; Su 2010; Kuksov & Liao 2023). For example, demand occurs over two periods, consumer values are uniformly distributed (Cui *et al.* 2014 allow for more generally distributed consumer values, Geng *et al.* 2007; Su 2010 have binary consumer values), consumers do not know their value in the advance period (preference uncertainty does not occur in Su 2010), and some agents are capable of reselling (either just speculators, or both speculators and consumers).

Reselling is defined to be the transfer of the good in period 2 between agents other than the seller, e.g. consumer to consumer or speculator to consumer. The seller has no control over *unauthorized reselling,* whereas the seller can fully regulate all trades with *authorized reselling.* The old-fashioned exchange of paper tickets between a speculator/scalper and a consumer is an example of unauthorized reselling. Electronic tickets enable authorized reselling because these tickets can only be exchanged with the technology the seller controls.

We define *transaction costs* to be factors outside of the seller's control that reduce consumer utility, whereas *transaction fees* are explicit charges the seller can collect that increase the effective price paid for the seller's product. Transaction costs lower the final utility a consumer receives from the product. For example, consumers may dislike the hassle of making an exchange, or experience anxiety over the validity of the product, or incur moral qualms for these trades. We consider both extremes in which consumers either incur very high transaction costs (they refuse to participate in reselling) or zero transaction costs (doing so is routine and easy). Given that speculators seek only to profit from trading, they incur no transaction cost to buy or sell. If the reselling market is authorized in the sense that the seller is able to fully regulate any and all transactions, then the seller can also impose transaction fees with each resale exchange. For example, the seller (or a third party agent authorized by the seller) may collect service fees, or convenience charges, or commissions on each trade. Previous studies of reselling do no give the seller the ability to earn transaction fees on exchanged goods.

The seller chooses the price p_i to sell units in period i. The seller can choose different prices across time, a practice that is often referred to as *dynamic pricing.* The seller can (but is not required to) restrict the number of units it offers for sale in either period. Su (2010) and Cui *et al.* (2014) focus on the comparison between static pricing (the seller can choose only one price) and dynamic pricing, whereas we begin with dynamic pricing as the main specification.

Efficient rationing determines who is able to purchase and who is able to sell. In particular, with efficient rationing, buyers are given priority in increasing order of their value for the good (i.e., the buyers with the highest value are first in line to purchase), and seller priority is in decreasing order of their offered price (i.e., the seller with the lowest price is first to sell). If sellers have the same price, then the seller with the lower value for the good sells first. (This tie breaking rule has no impact on the results.) Cui *et al.* (2014), Su (2010), and Kuksov & Liao (2023) also use efficient rationing, which resembles a double auction (McAfee 1992). Critically, there is competition among all of the agents with goods to sell in period 2, i.e., the seller, Figures/Timeline.png

Figure 1. Timeline of events

consumers, and speculators.

A consumer's utility is the value they receive for the good minus transaction costs, fees, and the price paid. Consumers and speculators are strategic - they select actions to maximize their utility given their correct expectations of the future actions of others and the resulting market prices. In period 1 each of the n_1 consumers who arrive in that period can choose to request a purchase or not. If they abstain from or are unable to purchase in period 1, they can purchase in period 2 if they desire (i.e., if their value is greater than the period 2 price) and if the good is available in that period. The value consumers receive from the good does not depend on which period they are able to acquire it, i.e., they are fully patient. Consequently, there is no notion that consumers receive any additional value from having a set plan well in advance of consumption.

The parameters and sequence of events are common knowledge to the consumers and the seller. All agents are risk-neutral. The seller's objective is to design the terms of trade to maximize expected revenue. The seller is able to commit to its selling mechanism, i.e. the prices and quantities in both periods, possibly contingent on observable outcomes. Section 5 discusses several extensions and variations to the model. Figure 1 displays a timeline of the actions.

4 Analysis of Selling Mechanisms

Several selling mechanisms are considered. They differ in whether consumer reselling occurs and in the seller's choices regarding prices and fees. The most basic is *spot selling*: the seller only sells in period 2 when consumers know their values. *Advance selling* is more sophisticated: the seller selects a period 1 price and quantity, but the seller may also sell some of its inventory in period 2. With spot selling there is no opportunity for reselling. With advance selling, as is generally assumed in other studies (e.g., Gale & Holmes 1993; Xie & Shugan 2001), consumer reselling does not occur either because consumer transaction costs from doing so are too high, or the seller is able to prevent reselling and chooses to do so. Next, we consider the seller's options when consumers have low transaction costs and are able to resell outside of the seller's control, a situation we refer to as *unauthorized reselling*. For example, a consumer who purchases a paper ticket to an event in period 1 can sell the ticket to another consumer in period 2. This is feasible when the seller is unwilling or unable to constrain such trades from happening, hence the label "unauthorized". Finally, we identify the seller's optimal selling mechanism (i.e., the mechanism that yields the highest revenue for the seller over all possible mechanisms). It happens to involve *authorized reselling* in which, due to enabling technology, the seller can regulate the transactions that occur among consumers, which, crucially, gives the seller the ability to charge transaction fees.

4.1 Spot Selling

A seller is not required to sell capacity over time. With spot selling the seller does not offer units to sell in advance (i.e., $q_1 = 0$). Instead, in period 2, the seller chooses a spot price, p_s, and a quantity $q_2 \leq q$ to offer for sale. Consumers choose to buy in period 2 after they observe their value for the good. In this situation, the seller's revenue in the spot period is

$$\pi_s = \min\{(1-p_s)\,n, q_2\}\,p_s.$$

We define *scarcity* to occur when some consumers who are willing to pay the period 2 price are not able to obtain a unit due to total demand exceeding supply, i.e., when scarcity occurs, so does rationing among the consumers. From π_s, it is not in the seller's interest to create scarcity in period 2 (i.e., the seller chooses $q_2 = q$): if there are more consumers willing to buy than the seller has inventory to serve (i.e., scarcity), the seller can increase its revenue by raising its price. Theorem 1 reports the seller's optimal price and some market outcomes.

Theorem 1. *With spot selling, the optimal price, p_s^*, quantity sold, q_s^*, and revenue, π_s^*, are given in the following table:*

Capacity	$\kappa < 1/2$	$1/2 \leq \kappa$
p_s^*	$1-\kappa$	$1/2$
q_s^*	κn	$n/2$
π_s^*	$\kappa\,(1-\kappa)\,n$	$n/4$

When supply is limited, $\kappa \leq 1/2$, the seller sells its entire capacity at the market clearing price to the consumers who are willing to pay the most. Hence, in these situations social welfare, which is the sum of the values of consumers who purchase, is maximized because all units are sold to the highest value customers. However, when supply is ample, $\kappa > 1/2$, the seller prefers to only sell a portion of its capacity. (Some consumers might describe this as scarcity because of the high price, but the high price prevents rationing, so based on our definition, scarcity does not occur.) Consequently, social welfare is not maximized (it would increase if the seller lowered the price and sold more).

Although actual sales only occur with spot selling in period 2, the seller could make units available in period 1 and achieve the same outcome. For example, if the seller offers p_s^* across both periods, consumers have no reason to purchase in advance and speculators do not enter the market, so the seller need not constrain the units available in advance.

4.2 Advance selling

With the advance selling mechanism the seller chooses a period 1 price with the goal to have consumers purchase in period 1. Consumer transaction costs are sufficiently high to preclude them from reselling. However, speculators are present and participate if they can earn a profit.

Advance selling creates a complex decision for the consumer. If they purchase in period 1, they must do so before they are certain of their value. Hence, it is possible that the price they pay in period 1 is higher than the value they observe for the good in period 2. If that were to happen, the consumer still consumes the good in period 2 because receiving some value for the good is better than wasting it entirely, and reselling is not an option. The consumer's alternative to purchasing in advance is to wait to period 2. Doing so allows the consumer to observe their value before purchasing, but the period 2 price might be greater than the period 1 price, and it is possible that even if they are willing to pay the period 2 price they may not be able to do so because scarcity causes rationing. Consumers account for all of these factors to make the best decision.

Consumers' expected utility from purchasing in period 1 is

$$u_1 = \mathbb{E}\left[V\right] - p_1 = 1/2 - p_1.$$

If they wait, their expected utility is

$$u_2 = \int_{p_2}^{1} (x - p_2)\, dx = (1 - p_2)^2 / 2$$

if they are confident they can purchase a unit at p_2 should they want to do so. If consumers could be rationed in period 2, then their expected utility from waiting to purchase would be less than u_2. However, it is never optimal for the seller to choose prices such that scarcity occurs. To explain, if rationing were to occur in period 2, the seller could strictly increase the period 2 price to eliminate any rationing, and doing so strictly increases the seller's revenue in period 2. Furthermore, consumer would be willing to pay even more in advance to avoid the higher period 2 price. Hence, the "cost" of creating scarcity (i.e., sufficiently low prices to create rationing) is not worth the benefit of doing so.

Comparing a consumer's utility from purchasing in period 1 with their expected utility from waiting to purchase, the most that customers are willing to pay in period 1 is

$$p_1(p_2) = u_1 - u_2 = p_2(2 - p_2)/2.$$

It follows that the seller's revenue with advance selling is

$$\pi_a = p_1 q_1 + p_2 q_2 = \left(p_2 - \frac{1}{2}p_2^2\right) q_1 + p_2 q_2,$$

where p_1 is a price consumers are willing to pay to purchase in period 1, q_1 is the number of units the seller chooses to sell in period 1, $q_1 \le \min\{n_1, q\}$, and q_2 is the number of units the seller makes available for sale in period 2:

$$q_2 \le \min\{(1 - p_2)(n - q_1), q - q_1\}.$$

Theorem 2. *With advance selling, the unique equilibrium optimal prices, p_1^* and p_2^*, quantities sold, q_1^* and q_2^*, and revenue, π_a^*, are given in the following table:*

Capacity	$\kappa \le \frac{1}{3}$	$\frac{1}{3} < \kappa \le \frac{1+\lambda_1}{3-\lambda_1}$	$\frac{1+\lambda_1}{3-\lambda_1} < \kappa \le \frac{1}{2-\lambda_1}$	$\frac{1}{2-\lambda_1} \le \kappa$
p_1^*	$\frac{1}{2}(1-\kappa)$	$\left(\frac{3-\kappa}{8}\right)(1+\kappa)$	$\frac{1}{2}\left(\frac{1-2\lambda_1+\kappa}{1-\lambda_1}\right)\left(\frac{1-\kappa}{1-\lambda_1}\right)$	$\frac{1}{2}\left(\frac{3-2\lambda_1}{2-\lambda_1}\right)\left(\frac{1}{2-\lambda_1}\right)$
p_2^*	$1-\kappa$	$\frac{1}{2}(1+\kappa)$	$\frac{1-\kappa}{1-\lambda_1}$	$\frac{1}{2-\lambda_1}$
q_1^*	0	$\frac{3\kappa-1}{1+\kappa}n$	$\lambda_1 n$	$\lambda_1 n$
q_2^*	κn	$\frac{(1-\kappa)^2}{1+\kappa}n$	$(\kappa - \lambda_1) n$	$\frac{(1-\lambda_1)^2}{2-\lambda_1}n$
π_a^*	$\kappa(1-\kappa) n$	$\frac{(1+\kappa)^2}{8}n$	$\frac{(1-\kappa)(2\kappa-\lambda_1(1+\kappa))}{2(1-\lambda_1)^2}n$	$\frac{1}{2(2-\lambda_1)}n$

Furthermore, all q_1^* units are purchased by early arriving consumers (i.e., speculators do not purchase).

Four cases emerge with advance selling which are illustrated in Figure 2. When supply is limited relative to demand, $\kappa < 1/3$ (case 1), advance selling provides no advantage over spot selling. Here, the seller can sell all of its supply at a high price, leaving no reason to sell to consumers in advance at a discount in an effort to increase sales, i.e., rather than choosing to sell in advance, the seller prefers to use spot selling. With the next three cases the seller has more capacity, $1/3 < \kappa$, and the seller actually does sell at least some of its inventory in advance. Whenever units are sold in advance, the seller must offer a discount ($p_1^* < p_2^*$) to motivate consumers to purchase early, but doing so allows the seller to sell more units. For example, in cases 2 and 3 ($\kappa < 1/(2 - \lambda_1)$) initial demand is ample and the seller sells all of its units, i.e., $q_1^* + q_2^* = q$.

AdvanceEq.png

Figure 2. Optimal advance selling in terms of the amount of capacity, κ, and the number of consumers arriving in advance, λ_1. Actual advance sales in period 1 only occur if capacity is sufficiently ample, $1/3 < \kappa$. The total number of units sold and the the quantity of advance sales differ across three regions.

(In case 2 only a portion of initial demand buys in advance, $q_1^* < n_1$, whereas in case 3 all early arriving customers purchase in advance, $q_1^* = n_1$.) In these situations, with the best spot selling strategy, the seller chooses to sell only a portion of its units ($1/2 < \kappa$), or sells all units ($\kappa < 1/2$) at a price that is less than the period 2 price with advance selling. Finally, in case 4, capacity is ample. The seller sells to all of the early arriving customers, but only sells a portion of the remaining inventory in period 2. Even though the seller does not sell its entire inventory, the seller still sells more units than with spot selling.

In sum, advance selling enables the seller to sell more units relative to what it would want to sell via spot selling. Increasing sales is most valuable when supply is relatively large, i.e. when some capacity could otherwise go unutilized. However, doing this requires selling at a discount for early purchases. If supply were limited (relative to demand), the seller would prefer to set a higher price and sell only to those customers who are willing to pay it. Put another way, rather than stating advance selling works *even when* supply is ample (relative to demand) (as suggested by Xie & Shugan 2001), it is better to state that advance selling is useful *only when* supply is ample.

Advance selling can provide a substantial boost to revenue over spot selling. For example, consider situations in which a reasonable portion of consumers arrive early (case 2 in Theorem 2). With $\kappa = 1/2$ and $1/3 < \lambda_1$, the seller earns 12.5% more revenue with advance selling than spot selling even though with either strategy the seller sells all of its inventory: with advance selling the seller sets an advance price that is less than the spot price but is able to sell for more that the spot price in period 2, increasing the average price paid over the spot price. With more capacity to sell, $\kappa = 2/3$ and $3/5 < \lambda_1$, the seller earns 38.9% more revenue with advance selling because advance selling allows the seller to sell its entire inventory but with spot selling the seller's best option is to sell only $1/2$ of its inventory.

Advance selling works for the seller, despite the need to give customers an initial discount, because consumers anticipate the limited supply in period 2 results in a high period 2 price. This encourages them to purchase in period 1 to secure a unit at a more reasonable price. While the seller must provide a discount to consumers to purchase in period 1, the uncertainty of early arriving customers in advance implies that by providing a sufficient discount, the seller is able to entice them to purchase in advance and thereby expand sales. If capacity is large enough such that the benefit from expanding sales outweighs the per-unit loss from the discount, the seller benefits from advance selling. But advance selling also comes with a cost. Some consumers who purchase in period 1 learn that they regret their purchase - their value of the good is less than the price they paid. Worse yet, in period 2 there are consumers who do not purchase that would gladly buy the unit from consumers who would rather sell. These potential trades are a lost opportunity, suggesting that advance selling might not be the best mechanism.

The effectiveness of advance selling has been identified in earlier work (e.g., Xie & Shugan 2001; DeGraba 1995), but these studies do not consider the possibility of reselling (either by consumers or speculators). Here, consumer transaction costs are taken to be high enough to prevent them from reselling. Speculators are able to resell, and the price increase from period 1 to period 2 suggests they could profit from doing so. But speculators can only resell if they are able to buy in period 1. With efficient rationing and the ability to limit the number of units it sells in advance, the seller is able to entirely prevent speculators from buying in period 1. This allows the seller to sell whatever remaining inventory it has at the start of period 2 for a price that is higher than its period 1 price. If speculators were able to purchase ahead of consumers in period 1 (which would violate efficient rationing), then the seller would be harmed because of the additional competition the speculators would create in period 2. In that case the seller would benefit if reselling were infeasible even for speculators. In sum, advance selling is a powerful tool for the seller with the important caveat that consumers cannot resell and speculators are prevented from purchasing units. The next section explores what happens when consumer reselling becomes viable (or speculators cannot be prevented from buying in advance).

4.3 Unauthorized Reselling

With advance selling it is presumed that reselling does not occur, either because the seller is able (and wants) to literally prevent it (e.g., via technology that links the sale of a unit to a person and prevents the buyer from transferring the unit to another person) and/or because transaction costs are sufficiently large to make any reselling unviable. Here, we consider how the market is changed by a frictionless reselling market, i.e., there are no transaction costs: units sold by individuals are deemed of equal quality to units sold by the seller, consumers have no aversion to trading, buyers and sellers can costlessly find each other and agree on an exchange price in which all of the value of the trade is retained by the buyer and seller, etc.

Let p_2 be the market price in period 2. Demand in period 2 at the market price is $d_2 = (1 - p_2)(n - q_1)$. If there are no speculators, then the total supply in period 2 is $s_2 = q_2 + p_2 q_1$, where $q_2 \leq q - q_1$ is the number of units the seller makes available on the market in period 2 and $p_2 q_1$ is the number of customers from period 1 who are willing to sell at that price. If there are speculators, then supply would be greater and the period 2 market price would be weakly lower, all else equal. For expositional clarity, we consider the equilibrium without speculators and later demonstrate that there cannot be an equilibrium with speculators.

As the market is well functioning, all transactions in period 2 occur at the same price, p_2, which clears the offered supply, s_2, and demand, d_2:

$$q_2 + p_2 q_1 = (1 - p_2)(n - q_1).$$

Hence, the clearing price is

$$p_2 = \frac{n - q_1 - q_2}{n}.$$

In period 1 consumers naturally account for the ability to sell in period 2 if they purchase in period 1 and discover that their value is lower than the market price. In particular, a consumer's utility from buying in period 1 is

$$u_1 = \frac{1}{2}(1 + p_2)(1 - p_2) + p_2^2 - p_1.$$

The first term is the expected utility conditional that the consumer wants to consume the good, while the second term is the expected utility conditional that the consumer sells the unit back to the market. Because p_2 clears the market, a consumer that wishes to sell at that price is able to do so. A consumer's expected utility from waiting to possibly purchase in period 2 after learning their true value for the good is $u_2 = \frac{1}{2}(1 - p_2)^2$: because supply clears demand, there is no scarcity, and a consumer knows that if they wait to period 2 to purchase then they are able to purchase if they are willing to pay the price p_2.

Given the utilities consumers earn from either buying in period 1 (with the option to sell in period 2) or waiting to purchase in period 2, we obtain a striking result: a consumer is willing to pay in period 1 as much as the anticipated period 2 price, i.e., $p_1 = p_2$. In other words, unlike with advance selling, the consumer in period 1 does not require a discount relative to the period 2 price. This is because consumers knows that they can always participate in the period 2 market. If their value is less than p_2, then they can sell for p_2, while if they do not have a unit and are willing to pay p_2, then they can buy at the price p_2. Consequently, buying in period 1 confers neither an advantage (e.g., access to scarce supply) or a disadvantage (the possibility of paying more for the good than it is valued). Hence, the period 1 price must match the period 2 price. The immediate consequence of the constant price path is that there are no opportunities for speculators - any entry by speculators would causes them to have to sell in period 2 for less than the period 1 purchase price, which means no speculator can enter the market and earn a profit.

The fact that consumers do not require a discount to purchase in period 1 when there is a reselling market would seem to suggest that this mechanism could be better for the seller than advance selling, which does require a discount to motivate consumers to purchase in advance. But in fact, the opposite is true. According to Theorem 3, with unauthorized reselling the seller can do no better than spot selling, which has already been confirmed to be (weakly) worse than advance selling.

Theorem 3. *With unauthorized reselling and either consumers or speculators (or both) have no transaction costs, the seller's optimal revenue, π_r^*, is equivalent to the revenue with spot selling, $\pi_r^* = \pi_s^*$.*

Some introspection reveals the intuition behind the failure of unauthorized reselling. Given that the reselling market matches supply with demand, all users with a value greater than p_2 consume a unit and no user with a value less than p_2 consumes a unit. This is true no matter if the user obtains the unit before learning their value (in period 1) or after learning their value (in period 2). Hence, all users that consume the unit pay the same single price p_2 and no user pays more than their value for the unit. This is exactly the final outcome as in spot selling. The only difference is the timing of when units are transferred to individuals. But the end conclusion of which users actually consume a unit and how much they pay remains the same. In sum, unauthorized reselling fails the seller because it lacks the ability to expand total sales, which is precisely the benefit of advance selling. With advance selling consumers take the gamble to buy in period 1 knowing that they might find themselves with a unit that is valued for less than they paid. Although this is an *ex post* bad outcome for the consumer, it does mean that the capacity is sold for some revenue, which works to the benefit of the seller. With unauthorized reselling, the market clears supply with demand, meaning that all units are consumed by consumers who don't regret their purchase (i.e., they value the good more than they pay for it). Consequently, units can be sold only to the consumers that ultimately learn they are willing to pay the offered price.

There are no profitable trades for speculators when consumers can purchase in period 1 and sell in period 2. The same result applies if speculators are able to purchase in period 1 ahead of all consumers (i.e., in violation of efficient rationing). If speculators had early access, then competitive entry would lower the period 2 price to again equal the period 1 price - if any speculator can make a profit, they enter, which puts downward pressure on the resale price until there are no further entry opportunities. In the end, even with speculators given priority to buy in advance, the same set of consumers are the ones that finally use the product and the seller earns no more than with spot selling.

The practical implication of Theorem 3 is that sellers should oppose unauthorized reselling. If consumers (or speculators) who purchase in advance have the opportunity to sell their units in the spot period, then the competition between those consumers and the seller essentially destroys the (considerable) advantages of selling in advance. The seller is effectively relegated to selling on the spot and earning less revenue than it otherwise would if the reselling market did not exist. This is consistent with the traditional view on reselling. The next section identifies what has changed to cause sellers to embrace reselling.

4.4 Authorized Reselling - The Optimal Mechanism

Advance selling (with no period 2 reselling market) can improve the seller's revenue relative to spot selling. Unfortunately, unauthorized reselling destroys the usefulness of advance selling, rending advance selling no better than spot selling. So what is the best selling mechanism for the seller? According to Theorem 4, the seller can do no better than advance selling combined with an authorized reselling market.

Theorem 4. *An authorized reselling market maximizes the seller's revenue. Consumers can buy in period 1 for p_1^* and in period 2 for p_2^*. Consumers who purchase in period 1 can sell their unit in period 2 for the market price p_2^*, but must also pay a transaction fee t^* to the seller. The seller makes its entire supply available for purchase in period 1, and all early arriving consumers (in period 1) are willing to buy, i.e., $q_1^* = n_1$, but speculators do not buy. In period 2, the price p_2^* clears the market, i.e., demand at price p_2^* equals the available supply provided by the seller (unsold units from period 1) and other consumers (i.e., there is no scarcity of supply). Prices, the transfer fee and the resulting seller profit are given in the following table:*

Capacity	$\kappa \leq \frac{1}{2-\lambda_1}$	$\frac{1}{2-\lambda_1} < \kappa$
p_1^*	$\frac{1}{2}\left(2 - 2\left(2-\lambda_1\right)\kappa + \left(3-2\lambda_1\right)\kappa^2\right)$	$\frac{3-2\lambda_1}{2(2-\lambda_1)^2}$
p_2^*	$1 - \kappa + \kappa\lambda_1$	$\frac{1}{2-\lambda_1}$
t^*	κ	$\frac{1}{2-\lambda_1}$
π^*	$\frac{1}{2}\kappa\left(2-\left(2-\lambda_1\right)\kappa\right)n$	$\frac{1}{2}\left(\frac{1}{2-\lambda_1}\right)n$

The reselling market is authorized in the sense that the seller is able to profit from each unit that is sold in the resale market. Through the regulation of its transaction fee, the seller controls the level of activity on the reselling market. When supply is ample (case 2), the seller has no interest to create additional supply in period 2, i.e. it does not want to compete with the consumers that purchased a unit in advance. In this case, the chosen transaction fee prevents any reselling. Here, the seller operates advance selling without a resale market, and the seller can do no better than this, i.e. advance selling is indeed the seller's optimal selling mechanism when there is sufficient supply.

The benefit of authorized reselling materializes when capacity is relatively scarce (case 1). In these situations, the reselling market increases the seller's revenue even over what can be achieved with advance selling. Because many units are sold in advance, the seller is left with relatively low inventory in period 2. Now it doesn't mind if consumers contribute to the supply in period 2, as long as the seller limits the amount

Rev_q.png

Figure 3. Comparison of revenues per unit of capacity, π/q, as a function of κ ($\lambda_1 = 0.75$)

they sell and takes a cut on each transaction, i.e., a positive transfer fee is needed to make this work for the seller. Note, the more demand arrives early (λ_1 increases), the broader the set of capacity conditions under which reselling improves revenue.

Although creating scarcity in period 2 would increase the amount consumers are willing to pay in period 1, with the optimal selling mechanism it never works to the advantage of the seller to do so. The cost of not selling units in period 2 always exceeds the benefit of the additional revenue that can be achieved with the higher period 1 price.

Figure 3 displays the seller's profit with each of the three mechanisms and across a range of capacity relative to demand, κ. When capacity is large relative to demand (κ is high, about 0.8 in the graph), spot selling performs poorly, the seller sets the transfer fee to eliminate the reselling market and advance selling captures all of the seller's potential. As κ decreases (demand increases relative to capacity), all mechanisms increase their revenue per unit of capacity, but advance selling begins to lose its relative advantage compared to spot selling. When κ is low ($\kappa < 1/3$), advance selling is no longer used. Nevertheless, the optimal mechanism continues to perform better than spot selling for all levels of capacity.

Recall, with advance selling the seller may choose to hold back some capacity to sell in period 2. The optimal mechanism does not need to do this because it retains the advantage of advance selling (avoiding wasted capacity) while it also overcomes the limitation of advance selling (possibly selling to the wrong consumer) by enabling units sold in period 1 to consumers with low values to be transferred to consumers in period 2 with higher values. In other words, reselling markets are most valuable to the seller when supply is limited (small κ) because the "expand sales" function of advance selling is not as important as ensuring the units are purchased by the consumers that value them the most.

To understand how the optimal mechanism improves upon spot selling, consider the case with limited supply, $\kappa \leq 1/2$, which is most favorable to spot selling. Consequently, with spot selling the seller sells all q units at the highest possible price. This maximizes social welfare because there are no remaining profitable trades - the q consumers with the highest value all receive a unit. It would seem that this is the best the seller could do, yet the optimal mechanism does strictly better. A comparison of the prices begins to explain why:

$$p_1^* < p_s^* < p_1^* + t^*$$

Like spot selling, with the optimal mechanism all units are sold. However, with the units initially purchased and not transferred to another consumer, the seller earns only p_1^* per unit, which is less than if those units were sold with exclusively a spot market. But with the units initially purchased and transferred, the seller earns $p_1^* + t$, which is more than the spot price. Hence, with the optimal mechanism the seller is able effectively to sell all q units at two prices rather than the single price which would occur with spot selling. This allows the seller to price discriminate across the consumers and net a higher revenue.

Although Theorem 4 describes a reselling market authorized by the seller, it is not necessary that the seller actually operates the market. The critical feature of this reselling market is that the seller is able to earn a portion of the revenue associated with each transaction. Obviously, the seller could do this if it owned the reselling market and possessed the technology to ensure that all trades were executed through this market. But it can also be achieved if the seller contracts with a third party that owns and operates the reselling market.

An authorized reselling market is the seller's best selling mechanism, but this does not mean the seller wants the reselling market to maximize the number of possible trades. The seller sets the transaction fee to regulate the number of trades on the market, thereby preventing the period 2 price from collapsing too low. Furthermore, the transaction fee provides the seller the ability to sell its capacity at multiple prices (as already discussed), which creates another source of revenue for the seller. In fact, according to Theorem 5, the revenue from the resale market can be substantial for the seller. This is particularly true when capacity is limited (low κ) and early arriving demand is high (high λ_1), as with popular concerts and sporting events.

Theorem 5. *Let π_t be the revenue that the seller collects from transaction fees, $\pi_t = (p_2^* - t^*)\, t^* \lambda_1 n$. Then, $\lim_{\kappa \to 0} \pi_t/\pi^* = \lambda_1$, $\lim_{\kappa \to 1} \pi_t/\pi^* = 0$ and π_t/π^* decreases in κ.*

Although reselling requires the seller to have access to some transaction fee, if for some reason the seller must charge less than the optimal transaction fee, it may still earn a substantial portion of its optimal revenue: if the transaction fee is $3/4^{th}$ of the optimal, then revenue is about 94% of optimal in the worst case. In other words, the seller's revenue function is relatively flat about the optimal transaction fee. Similarly, if the optimal transaction fee is imposed on the market but the seller collects only half of it, then the seller's revenue is still at least 75.9% of optimal. This is a practically important finding because it illustrates that sellers can benefit from authorized reselling even in situations in which sellers use third-party resellers, such as Ticketmaster and Seat Geek, and must give up significant revenue to enter into revenue-sharing agreements with them. That said, as already discussed, the seller does not do well if there is a reselling market with no transaction fee (i.e., an unauthorized reselling market). In that case the seller is relegated to the mere revenue of simple spot selling.

The potential activity of speculators raises one concern with the implementation of a reselling market. Speculators are consumers who know they have zero value for the good yet nevertheless trade with the goal to earn income. It has been suggested that speculators provide a useful function for the seller if the market is inefficient (Su 2010, Cui *et al.* 2014). However, speculators are unable to participate (profitably) and have no useful role to play in an efficient reselling market. To explain, speculators are at a disadvantage relative to the other consumers because they are not willing to pay the seller's initial price, p_1^* (because $p_1^* > p_2^* - t^*$). Only consumers that know they will receive some value for the good are willing to pay p_1^*. Thus, the seller need not be concerned with the possibility that speculators may influence the market and the seller surely does not need speculators to maximize its revenue.

Another concern with reselling markets is the potential to create competition between the seller and its previous customers. This is not an issue. It is optimal for the seller to maximize (not minimize) the number of consumers who purchase in period 1. To explain, each unit sold in period 1 creates a potential future competitor, which lowers p_2. This logic is intuitive, but incomplete. A seller can either sell a unit to a consumer in period 1 or keep the unit itself. If the unit is sold to the consumer, then the consumer sells it in period 2 only if the price is sufficiently high because the consumer has some value for the good, i.e., the consumer might not value the unit all that much, but that doesn't mean the consumer would be willing to sell for a pittance. In contrast, the seller has no value for the good in period 2, so the seller prefers any price over not selling the item. This means that the seller has a stronger incentive to sell in perid 2 than the consumer, so it is the seller that puts more downward pressure on prices. In sum, the competition the seller should fear is the future competition from itself (who is desperate to unload the units) rather than from consumers (who do not experience the same level of pressure to sell). That is why it is best for the seller to sell as much as possible in period 1.

5 Discussion

Our main model is intentionally parsimonious to achieve clean insights, but several extensions are worth discussing.

Fixed pricing. In models that do not consider transaction fees, previous work assumes the seller is restricted to set the same price across time, $p_1 = p_2$. With rigid pricing, the entry of speculators can be helpful for the seller by effectively mimicking dynamic pricing (e.g., Su 2010; Cui *et al.* 2014). Our model confirms this result. To explain, say consumer transaction costs are high, so they cannot resell, the seller cannot prevent unauthorized reselling (e.g., paper tickets), and initial demand is strictly less than capacity ($n_1 < q$) but also large enough so that advance selling increases revenue over spot selling. Under these conditions a seller benefits from selling at the price p in advance to all period 1 consumers and the seller's remaining units to speculators. Given that speculators take the seller's remaining inventory, in period 2 the seller has nothing left to sell at the price p. However, the speculators can sell at a price higher than p, and earn a profit, which is why they are willing to buy the seller's remaining inventory in period 1. If the seller had retained those units, then the seller would have to offer them for the lower price of p in period 2. So selling to speculators in period 1 raises the period 2 price (because the seller is constrained to retain the same price p over the two periods). As the period 2 price increases (relative to the case without speculators), consumers in period 1 are willing to pay more to obtain a unit (to avoid the higher period 2 price). And selling all of its inventory for a higher period 1 price clearly benefits the seller. In effect, speculators allow the seller to indirectly implement dynamic pricing. However, the stipulated conditions all need to hold for the seller to benefit from speculators. If the seller is not restricted to a single price, or initial consumer demand is ample enough to allow the seller to sell all of its inventory to them in advance ($q < n_1$) or consumers can resell, then speculators are of no use to the seller. Moreover, authorized reselling remains optimal even if the seller is indeed forced to sell at the same price across time.

Alternative rationing rules. Most of our results are robust to alternative rationing rules and do not hinge on the implementation of efficient rationing. For example, with spot selling the seller prefers to increase the price rather than to cause rationing. Similarily, with an authorized reselling market, all consumers who want to sell at the price p_2 can sell, and all buyers who want to purchase at that price can as well, so rationing does not occur. The same applies with unauthorized reselling because the market clears supply and demand. However, the particular rationing rule may be relevant with advance selling. Recall, with advance selling units are sold in advance at a discount. If all units are sold in advance, because early arriving demand is ample, then there are no units to sell in the spot period, so rationing does not impact the market. But if the seller is unable to sell all of the units in advance, then the particular rationing rule is important. As argued in Section 4.2, given efficient rationing, the seller can limit the supply available in advance and doing so prevents speculators from obtaining units. Keeping speculators out of the market is important, otherwise the participation lowers the period 2 price and harms the seller. With other rationing rules, it may not be possible for the seller to exclude speculators. In sum, we have established that the benefits of advance selling can disappear if consumers have low transaction costs, and the same could be true with alternative rationing rules.

Intermediate levels of transaction costs. The efficiency of the reselling market contributes to the sharp result in Theorem 3. In practice, a reselling market would involve some transaction costs. Finding another buyer for a ticket in period 2 does not occur without effort, and a period 2 buyer may have somewhat of a preference to obtain a unit directly from the seller rather than to purchase it from another consumer. If such transaction costs exist, and if those costs are substantial enough, then the reselling market is eliminated, leaving the seller only with the option to advance sell. In other words, as transaction costs increase, the negative effect of unauthorized reselling decreases, but in the limit, with extremely high transaction costs, the seller is left with advance selling.

Distributions of consumer valuations. The reselling literature typically assumes consumer values are uniformly distributed (e.g., Kuksov & Liao 2023) or follow a simpler two-point distribution (e.g., Geng *et al.* 2007; Su 2010). Consumer values are also uniformly distributed in our model and this allows for closed form expressions for prices, quantities, and revenues. However, none of our results require specific assumptions on the distribution and all hold qualitatively for any general distribution sufficiently well-behaved to yield unimodal revenue in prices.

Incentive to invest in capacity. Throughout our analysis we have taken capacity as fixed. However, there may be opportunities, especially in the long run, for the seller to adjust capacity. With spot selling, seller revenue increases in capacity only up to a point, i.e., $\kappa < 1/2$, and afterwards additional capacity provides no increase in revenue. With advance selling, the motivation to expand capacity is stronger. In particular, when $1/2 < \kappa < 1/(2 - \lambda_1)$, additional capacity provides no value to the spot selling seller but does increase revenue with advance selling. More capacity is helpful to the seller because advance selling allows the firm to expand sales to ensure that capacity is sold. It follows that expanding capacity is useful to the seller in a broader set of circumstances when early arriving demand increases (i.e., $1/(2 - \lambda_1)$ increases in λ_1). Furthermore, whenever there is an incentive to expand capacity with advance selling, there is also an incentive to do so with authorized reselling.

6 Conclusion

This paper resolves an enigma: why is it that in the past sellers opposed reselling, but now, not only do they support reselling markets, they even join them by signing agreements with third-party resellers? The reason, according to our model, is technology. With technology the seller can control exchanges and profit from them via transaction fees. This control is crucial for the success of such markets: without the ability to manage exchanges and charge transaction fees, such as with reselling markets operating offline with unidentifiable paper tickets, unauthorized reselling harms the seller by destroying all the benefit that can be achieved by selling in advance. This explains the hostility towards reselling in the past. But the emergence of new technology that allows the regulation of exchanges and collection of transaction fees allows the seller not only to restore revenue to the advance selling level, but also to achieve even higher revenue, in particular when capacity is scarce (relative to demand), as would occur with popular events.. In fact, while unauthorized reselling is detrimental to sellers, authorized reselling with an optimally chosen transfer fee is the best mechanism the seller can hope for. Even suboptimal transaction fees lead to near-optimal revenue, implying that while it is important to charge a fee, sellers have significant flexibility in the fee they select, which may help in their negotiations with third-party resellers.

Although the negative impact of speculators is often feared, we find that in an efficient reselling market speculators do not participate and should be of no concern. Along with helping the seller achieve optimal revenue, the presence of consumers who are willing to resell makes it unprofitable for speculators to enter the market. Clearly, authorized reselling can only be effective if consumers are able to trade without incurring excessive transaction costs (e.g., disutility associated with the time and effort to participate in the market, the perception of engaging in an unethical activity, and/or the fear that a trade may not be legitimate.) However, both the official sponsorship of the exchanges and the use of information technology generally reduce transaction costs, making authorized reselling feasible and highly profitable. This is consistent with the observation that instead of using technology to prevent resale, many sellers of perishable capacity (e.g., sports teams) now use this technology to actively encourage reselling among their consumers.

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A Proofs of Theorems

Proof of Theorem 1. Spot selling occurs when the seller does advance selling without any advanced sales, $q_1 = 0$. Hence, spot selling is a special case of advance selling, and the two mechanisms can be analyzed in the same framework - see (2). $\square$

Proof of Theorem 2. A consumer who purchases in period 1 earns an expected utility of $1/2 - p_1$. Alternatively, the consumer could wait to attempt to purchase in period 2 at price p_2. The expected value of waiting given a period 2 price of p_2 is $(1-p_2)^2/2$: there is a $1-p_2$ probability the consumer has a utility high enough to be willing to purchase at p_2 and conditional on that, the expected gain in utility over the price paid is $(1-p_2)/2$. Hence, when consumers expect p_2 to be the period 2 price, the most the seller can charge consumers in period 1 is

$$p_1 \leq \frac{1}{2} - \frac{(1-p_2)^2}{2} = \frac{1}{2} p_2 (2-p_2).$$

In period 2 available demand is $n - q_1$ and available supply is $q - q_1$. (Recall, there is no reselling, so the supply is entirely from the seller's period 1 inventory that is not sold.) The quantity sold in period 2 is $q_2 \leq \min\{(1-p_2)(n-q_1), q-q_1\}$. For any p_2, it is optimal to maximize q_2, so revenue is

$$\pi_a(p_2, q_1) = \left(p_2 - \frac{1}{2}p_2^2\right) q_1 + p_2 \min\{(1-p_2)(n-q_1), q-q_1\}$$

Demand is less than supply when

$$(1-p_2)(n-q_1) < q - q_1 \Longleftrightarrow q_1 < \frac{q-(1-p_2)n}{p_2},$$

otherwise it is the supply constraint that binds.

Revenue is

$$\pi_a(p_2, q_1) = \left(p_2 - \frac{1}{2}p_2^2\right) q_1 + \begin{cases} p_2(1-p_2)(n-q_1) & q_1 \le \frac{q-(1-p_2)n}{p_2} \\ p_2(q-q_1) & \text{otherwise} \end{cases}$$

$$= \begin{cases} p_2(1-p_2)n + \frac{1}{2}p_2^2 q_1 & q_1 \le \frac{q-(1-p_2)n}{p_2} \\ p_2 q - \frac{1}{2}p_2^2 q_1 & \text{otherwise} \end{cases}$$

If $p_2 < 1 - q/n$ then the second case of the revenue function (demand is greater than supply) applies for all $q_1 \ge 0$. For these values of p_2, revenue is strictly decreasing in q_1, which implies $q_1 = 0$ is best. However, given $q_1 = 0$, revenue is strictly increasing in p_2, which implies $p_2 < 1 - q/n$ cannot be optimal. Hence, the optimal solution must have $p_2 \ge 1 - q/n$, i.e., demand is (weakly) less than supply.

Given $p_2 \ge 1 - q/n$, profit is strictly increasing in q_1. It follows that the optimal period 1 sales are

$$q_1^* = \min\left\{\frac{q-(1-p_2)n}{p_2}, n_1\right\}$$

which when combined with $p_2 \ge 1 - q/n$ yields

$$q_1^* = \begin{cases} \frac{q-(1-p_2)n}{p_2} & \frac{n-q}{n} \le p_2 \le \frac{n-q}{n-n_1} \\ n_1 & \frac{n-q}{n-n_1} < p_2 \le 1 \end{cases}$$

and the revenue function can now be written as

$$\pi_a(p_2) = \begin{cases} \frac{1}{2}p_2(q+(1-p_2)n) & \frac{n-q}{n} \le p_2 \le \frac{n-q}{n-n_1} \\ p_2(1-p_2)n + \frac{1}{2}p_2^2 n_1 & \frac{n-q}{n-n_1} < p_2 \le 1 \end{cases}$$

Revenue is not differentiable at the boundary between the two regions, but is overall concave because the kink is downward sloping at the boundary. The unconstrained optimal price for the first and second regions are

$$p_2' = \frac{q+n}{2n} = \frac{1+\kappa}{2}$$

$$p_2'' = \frac{n}{2n-n_1} = \frac{1}{2-\lambda_1}$$

Define $\hat{p}_2$ to be the price that marks the boundary between the two regions of the revenue function, i.e., the price at which the optimal period 1 sales exactly equals the available period 1 demand,

$$\hat{p}_2 = \frac{n - q}{n - n_1} = \frac{1 - \kappa}{1 - \lambda_1}$$

The optimal period 2 price is the lower bound of the first region when $p_2' \leq \frac{n-q}{n} = 1 - \kappa$, which can be written as $\kappa \leq 1/3$. In this case the period 2 price is selected to clear the available supply and no units are sold in advance, $q_1^* = 0$. In other words, spot selling is optimal - supply is sufficiently limited that the best option for the seller is to sell that supply on the spot market to the consumer who know they are willing to pay the clearing price.

The optimal period 2 price is p_2', which is interior to the first region, when $1 - \kappa < p_2' < \hat{p}_2$, i.e., $1/3 < \kappa$ and

$$\frac{3\kappa - 1}{1 + \kappa} < \lambda_1$$

In this case some units are sold in period 1, but fewer than could be sold because period 1 demand is ample, i.e., $q_1^* < n_1$.

The optimal period 2 price is $\hat{p}_2$, which is the boundary between the two regions, when $p_2'' \leq \hat{p}_2 \leq p_2'$, i.e., $1/3 < \kappa$ and

$$\frac{2\kappa - 1}{\kappa} \leq \lambda_1 \leq \frac{3\kappa - 1}{1 + \kappa}$$

In this case $q_1^* = n_1$ and all remaining units are sold in period 2.

The optimal period 2 price is p_2'', which is interior to the second region of the profit function, when $\hat{p}_2 < p_2''$, i.e., $1/3 < \kappa$ and

$$\lambda_1 < \frac{2\kappa - 1}{\kappa}$$

In this case $q_1^* = n_1$ but some units in period 2 remain unsold.

The combination of these conditions yields the optimal p_2^* as stated in Theorem 2. Substitution of p_2^* into the expressions for p_1, q_1 and π_a, obtains the remaining results. □

Proof of Theorem 3. In period 2, because the period 1 buyers have no transaction costs, they offer $p_2 q_1$ supply to the market. Equating period 2 offered supply and demand, we have $q_2 + p_2 q_1 = (1 - p_2)(n - q_1)$. Express the unconstrained period 2 quantity, $\hat{q}_2$, in terms of the period 2 price: $\hat{q}_2 = (1 - p_2) n - q_1$. The period 2 quantity from the seller cannot be negative, $q_2 \geq 0$, and cannot exceed the amount of supply the seller has available, $q_2 \leq q - q_1$.

The period 2 price is maximized when the seller offers no supply in period 2, i.e., $q_2 = 0$. Let p_H be the upper bound on the period 2 price that occurs when $q_2 = 0$, $p_H = 1 - q_1/n$, i.e., this is the period 2 price that equates the supply offered by the period 1 customers, $p_2 q_1$, with demand from the period 2 buyers. As p_H is an upper bound on the period 2 price, in any equilibrium$p_2 \leq p_H$. Note that $p_H > 0$ holds because $q_1 < n$. Let p_L be the lower bound on the period 2 price, i.e., $p_2 \geq p_L$.

The period 2 price is minimized when the seller offers all remaining inventory to the market, i.e., $q_2 = q - q_1$. Let p_L be this lower bound on the period 2 equilibrium price:

$$p_L = p_2 (q - q_1) = \frac{n-q}{n}.$$

In sum, the period 2 price can range on the interval $[p_L, p_H]$. The upper bound p_H is decreasing in q_1 and ranges on the interval $[1 - n_1/n, 1]$.

Total revenue

$$\begin{aligned}
\pi(q_1, p_2) &= p_1 q_1 + p_2 q_2 \\
&= p_2 q_1 + p_2 ((1 - p_2) n - q_1) \\
&= p_2 (1 - p_2) n \\
\text{s.t.} \quad & p_L \leq p_2 \leq p_H \\
& p_L = \frac{n-q}{n} \\
& p_H = \frac{n-q_1}{n} \\
& \frac{n-n_1}{n} \leq p_H \leq 1 \\
& 0 \leq q_1 \leq q
\end{aligned}$$

In all cases revenue is increasing in q_1. Recall, the $p_2 \leq p_H$ constraint is decreasing in q_1. Hence, with the optimal solution either the $p_L \leq p_2$ constraint binds, or the $p_2 \leq p_H$ constraint binds. Unconstrained revenue is maximized with $p_2 = 1/2$. Hence, with the optimal solution the $p_L \leq p_2$ constraint binds if

$$\frac{n-q}{n} \geq \frac{1}{2} \Longleftrightarrow \kappa \leq 1/2$$

and otherwise it is the $p_2 \leq p_H$ constraint that binds. Hence, the analysis of the optimal decisions can be divided into two regions: $\kappa \geq 1/2$ and $\kappa < 1/2$.

Region I: $\kappa \geq 1/2$. The optimal solution has the constraint $p_2 \leq p_H$ bind. That means that the seller does not sell anything in period 2, $q_2 = 0$. In this case then $q_1 = (1 - p_2) n$. The revenue function is then $\pi_I = p_2 (1 - p_2) n$. The unconstrained optimal price is $p_2 = 1/2$, which yields an unconstrained period 1 quantity of $q_1 = n/2$. It satisfies the $q_1 \leq q$ constraint when $\kappa \geq 1/2$, which holds in this region. Hence, in this region the optimal price is $p_2^* = 1/2$ and the resulting revenue is $\pi_I = n/4$.

Region II: $\kappa < 1/2$. The optimal solution has the $p_2 \geq p_L$ constraint bind, which implies $q_1 = 0$ and $p_2 = p_L = 1 - \kappa$. The resulting revenue is $\pi_{II} = n\,(1-\kappa)\,\kappa$, which is spot selling. □

. From the Revelation Principle (Myerson, 1981), the optimal mechanism belongs within the set of truth-inducing mechanisms - all agents report their type (i.e., their value for the good, when that is known), decisions are made based on the reported types, and truthful reporting of one's type is a Nash equilibrium strategy. To summarize the structure of the mechanism, there is a single price for selling units in period 1 to consumers because all consumers are identical in this period. Speculators do not purchase in period 1 because the seller does not need speculators. In period 2 there are two prices, one to sell units to consumers without a unit and one to buy units from consumers who prefer to relinquish their unit for the payment rather than consume the unit. The period 2 prices need not be the same. As the seller is the designer of the mechanism, the seller establishes the prices and controls all transactions. If the period 2 prices are not identical the seller either earns a profit on each transaction (if the price to sell is lower than the price to buy), otherwise the seller subsidizes period 2 transfers between consumers. That said, in period 2 there is no requirement that the number of units purchased by consumers is the same as the number sold by consumers (or even the total supply of units that can be acquired). Furthermore, rationing in period 2 is feasible (i.e., rationing occurs when demand in period 2 given reported types exceeds the supply of units). Any rationing, if it occurs, is correctly anticipated and accounted for by all parties in their decisions. The remainder of the proof formalizes these ideas and validates the proposed mechanism.

In period 1 there are two types of potential buyers, consumers and speculators and either a unit is transferred to each of these agents or not. In period 2 there are two types of consumers, ones that purchased in period 1, which are referred to as the "resellers" and those that did not, which are referred to as the "buyers". There can also be speculators in period 2 (if they received a unit in period 1).

The period 1 each agent either reports they are a "consumer" or they are a "speculator" - we use quotes to designate a report rather than ground truth.Consumers only know they have a value that is uniformly distributed $U[0,1]$ whereas speculators know they have zero value for the good. Among those that report they are "consumers", either a unit is transferred to them for the price p_1 or they do not receive a unit. A unit could be transferred to any agent that reports they are a "speculator" for a different price p_1'. Let q_1, $q_1 \leq q$, be the number of units transferred to "consumers". There may also be some number of units transferred to "speculators".

We first rule out the use of speculators in any optimal mechanism. Say there exists an optimal mechanism in which speculators pay p_1' in period 1 and receive p_2' in period 2, where $p_1' \leq p_2'$. The seller will not pay p_2' to a speculator for their unit unless the seller is able to receive at least p_2' for the unit. But if the seller can receive p_2' for the unit, the seller is better off not selling the unit to the speculator in period 1 for p_1', and simply selling the unit for at least p_2' in period 2. Hence, an optimal mechanism cannot include transfers to speculators. Thus, any agent reporting to be a "speculator" in period 1 receives no unit. As "speculators" do not receive a unit in period 1, consumers have no reason to claim they are a "speculator". We later confirm that speculators have no reason to claim to be "consumers".

In period 2 resellers either keep their unit or return it to the seller for some payment. Buyers can receive a unit for some fee. Consumers are distinguished only by their reported value and whether they possess a unit or not. Thus, in a truth inducing mechanism, there can only be a single price offered to buyers, p_2: if there were two or more prices assigned to the reported values from the buyers then they all would have an incentive to report a value associated with the lowest price.[1] Similarly, there can only be a single price offered to the resellers. However, those two prices need not be identical. Hence, without loss of generality, let $p_2 - t$ be what each reseller receives for units they return to the seller, where t is referred to as the transfer fee in period 2. (It is sufficient to assume $t \geq 0$, because if $t < 0$, then the seller is charging consumers to return their item, which none will do.) With this structure a seller earns a profit of t in period 2 from each unit that is purchased from resellers for $p_2 - t$ and sold to a buyer for p_2. There is no requirement that the number of units purchased from resellers in period 2 matches the number of units sold.

The supply of units in period 2 comes from units that the seller does not transfer in period 1 and those that period 2 resellers are willing to relinguish,

$$s_2 \quad = \quad q - q_1 + (p_2 - t)\, q_1$$

Demand in period 2 comes from all consumers who were not able to receive a unit in period 1,

$$d_2 = (n - q_1)\,(1 - p_2)$$

Let q_2 be the total number of units the seller makes available to the market in period 2, where clearly $q_2 \leq s_2$.

Given efficient rationing, consumers with the highest values in period 2 get the unit. Let $\hat{v}$ be the lowest consumer value that would receive a unit if there were q_2 units available:

$$(1 - \hat{v})\,(n - q_1) = q_2$$

[1]This applies independent of the rationing rule. For example, with efficient rationing units are sold in decreasing order of their valuation. The most a seller can obtain is the highest price offered. Hence, there is no incentive to offer any price lower than that highest price, i.e., a single price maximizes the seller's revenue.

or

$$\hat{v} = 1 - \frac{q_2}{n - q_1}$$

If $p_2 < \hat{v}$, then period 2 demand at price p_2 exceeds supply (i.e., there is rationing), and the entire offered supply is sold in the market to all consumers with values $\hat{v}$ or higher. Otherwise, supply (weekly) exceeds demand (i.e., no rationing), all customers who want to purchase at the price p_2 are able to do so, and some units remain unsold. Hence, all consumers with values v or larger, where $v = \max\{\hat{v}, p_2\}$, purchase a unit in period 2 for price p_2.

The period 1 price depends on what the customer is willing to pay. In period 1 customers pay p_1 and then with probability $p_2 - t$ they return their unit for a payment of $p_2 - t$. So the customer's net utility is

$$\begin{aligned} u_1 &= \tfrac{1}{2}(1 + p_2 - t)(1 - (p_2 - t)) + (p_2 - t)^2 - p_1 \\ &= \tfrac{1}{2}\left(1 + (p_2 - t)^2\right) - p_1 \end{aligned}$$

A customer that waits is willing to buy at price p_2 with probability $1 - v$. The customer earns utility

$$u_2 = (1 - v)\left(\frac{1 + v}{2} - p_2\right)$$

So the customer is willing to pay

$$p_1 = \tfrac{1}{2}\left[1 + (p_2 - t)^2 - (1 - v)(1 + v - 2p_2)\right]$$

A speculator in period 1 is not willing to claim to be a "consumer" because a consumer with a positive value is willing to pay at most p_1, so a speculator, knowing that their value is 0, surely is not willing to pay p_1.

The seller's total revenue is

$$\pi = p_1 q_1 - (p_2 - t)^2 q_1 + p_2 \min\{q_2, d_2\}$$

and so the seller's optimization problem is

$$
\begin{aligned}
\pi \quad &= \quad \tfrac{1}{2}\left[1-(p_2-t)^2-(1-v)(1+v-2p_2)\right]q_1+p_2\min\{q_2,d_2\} \\
\text{s.t.} \quad & \qquad d_2=(n-q_1)(1-p_2) \\
& \qquad s_2=q-q_1+(p_2-t)\,q_1 \\
& \qquad q_2\le s_2 \\
& \qquad q_1\le n_1 \\
& \qquad v=\max\left\{1-\tfrac{q_2}{n-q_1},p_2\right\} \\
& \qquad t\le p_2
\end{aligned}
$$

where $p_1 q_1$ is the revenue from selling q_1 units and $(p_2-t)^2 q_1$ is the cost of buying back the units in period 2 from the period 1 buyers with low values.

The optimization of revenue can be divided into two parts, cases in which $t=p_2$ is optimal and cases in which $t<p_2$ is optimal. If $t=p_2$ is optimal, then consumers will not resell in period 2. This is equivalent to advance selling with no reselling, which is analyzed in the proof of Theorem 2.

Alternatively, the seller can choose a transaction fee such that some consumers may wish to sell in period 2: $t<p_2$. First establish that with any optimal solution (given $t<p_2$), the seller makes the full supply available to the market, i.e. $q_2=s_2$. In other words, if the seller encourages consumers to create supply in period 2 (i.e., to resell), then the seller makes its entire period 2 supply available to the market in period 2. If $q_2<s_2$, then t can be increased without changing v. For a fixed v, profit is increasing in t. So if $q_2<s_2$, profit can be increased by increasing t, which is feasible whenever $t<p_2$. If follows that if $t<p_2$, then any optimal solution must have $q_2=s_2$.

Assuming $t<p_2$ and given $q_2=s_2$,

$$\hat{v}=1-\frac{q_2}{n-q_1}=1-\frac{q-q_1+(p_2-t)\,q_1}{n-q_1}=\frac{n-q-(p_2-t)\,q_1}{n-q_1}$$

$$v=\begin{cases}\frac{n-q-(p_2-t)q_1}{n-q_1} & p_2<\frac{n-q+tq_1}{n}\\ p_2 & \text{otherwise}\end{cases}$$

and the revenue function is

$$\begin{aligned}
\pi\left(t<p_2\right) &= p_1 q_1-\left(p_2-t\right)^2 q_1+p_2 s_2 \\
&= \frac{1}{2}\left[1-\left(p_2-t\right)^2-(1-v)\left(1+v-2 p_2\right)\right] q_1+\begin{cases} p_2 s_2 & p_2<\frac{n-q+t q_1}{n} \\ \left(n-q_1\right) p_2\left(1-p_2\right) & \text{otherwise}\end{cases} \\
&= \frac{1}{2}\left[\left(1-\left(p_2-t\right)^2\right)\right] q_1+\begin{cases}\frac{2 n^2 p_2+q_1\left(q+q_1\left(1+p_2-t\right)-2 n\left(1+p_2\right)\right)}{2\left(n-q_1\right)^2}\left(q-\left(1-\left(p_2-t\right)\right) q_1\right) & p_2<\frac{n-q+t q_1}{n} \\ n p_2\left(1-p_2\right)-\frac{1}{2}\left(1-p_2^2\right) q_1 & \text{otherwise}\end{cases}
\end{aligned}$$

In the first condition, $p_2<\left(n-q+t q_1\right)/n$, the supply of units in period 2 is less than the demand $\left(s_2<d_2\right)$, which creates some rationing, otherwise demand is unconstrained by supply.

The seller's optimization problem is

$$\begin{aligned}
\max \quad & \pi\left(t<p_2\right) \\
\text{s.t.} \quad & q_1 \leq n_1 \\
& t \leq p_2
\end{aligned}$$

The revenue function has two regimes: (i) $s_2<d_2$ and (ii) $d_2 \leq s_2$. Consider the first regime, $s_2<d_2$, which corresponds to period 2 prices such that

$$p_2<\frac{n-q+t q_1}{n} \tag{1}$$

Let $b=p_2-t$. The profit function is

$$\pi=\frac{1}{2}\left[\left(1-b^2\right)\right] q_1+\frac{2 n^2 p_2+q_1\left(q+q_1(1+b)-2 n\left(1+p_2\right)\right)}{2\left(n-q_1\right)^2}\left(q-(1-b) q_1\right)$$

subject to the constraint (1).Differentiate with respect to p_2,

$$\frac{d \pi}{d p_2}=\frac{\left(q-(1-b) q_1\right) n}{\left(n-q_1\right)^2}\left(n-q_1\right)$$

Profit is strictly increasing in p_2 in this region (for any fixed b). Hence, the optimal solution is not in the first regime. It is either at the boundary between the two regimes, $s_2=d_2$, or in the interior of the second regime, $d_2<s_2$. Thus, rationing in period 2 is never optimal, despite the fact that rationing increases the price consumers are willing to pay in period 1. Given that rationing is never optimal (conditional that buyers report their type truthfully in period 2), all buyers with values p_2 or greater announce their value and pay p_2, and all buyers with values less than p_2 also correctly report their value (they don't have an incentive to declare a higher value because then they would purchase the item for more than they would value it). So given that units are sold for p_2 in period 2 without rationing, buyers indeed report their type truthfully.

Given that rationing in period 2 does not occur, $v = p_2$, the period 1 price that can be charged is

$$p_1 = \frac{1}{2}\left[t^2 + 2p_2\left(1-t\right)\right]$$

Note that $p_2 - t < p_1 < p_2$ (given that $t < p_2$).

The seller's optimization problem can now be written as

$$\begin{aligned} \max \pi\left(t < p_2\right) &= \tfrac{1}{2}\left(p_2^2 - \left(p_2 - t\right)^2\right) q_1 + np_2\left(1-p_2\right) \\ \text{s.t.} \qquad & q_1 \leq n_1 \\ & \tfrac{n-q+tq_1}{n} \leq p_2 \\ & t < p_2 \end{aligned}$$

If

$$\frac{2\kappa - 1}{\kappa} < \lambda_1 \leq \kappa$$

then the optimal solution is

$$t' = \frac{q}{n} = \kappa$$

$$q_1' = n_1$$

$$p_2' = \frac{n - q + \frac{q}{n} n_1}{n} = 1 - \kappa + \kappa\delta$$

$$s_2 = q - \left(2 - \delta\right)\kappa n_1 = \left(1 - \lambda_1\right)^2 \kappa n$$

$$d_2 = \left(1 - \lambda_1\right)^2 \kappa n$$

$$p_1' = 1 - \left(2 - \lambda_1\right)\kappa + \left(\tfrac{3}{2} - \lambda_1\right)\kappa^2$$

$$\pi'\left(t < p_2\right) = \frac{q\left(2n^2 - 2nq + n_1 q\right)}{2n^2} = \frac{1}{2}\left(\delta - \frac{2\kappa - 1}{\kappa} + \frac{1}{\kappa}\right)\kappa^2 n$$

The seller sells all units across the two periods and a reselling market exists.

If

$$\lambda_1 < \frac{2\kappa - 1}{\kappa}$$

then the $p_2 = t$ constraint binds and the seller is in "advance selling" mode. Note, this range does not exist if $\kappa < 1/2$.

Note that the marginal increase in revenue with respect to capacity is increasing in λ_1, i.e., as period 1 demand becomes a larger portion of total demand, there is a stronger marginal incentive to increase capacity.

Combining the cases with $t < p_2$ and $t = p_2$ yields:

if $\kappa > 1/2$

$$q_1' = n_1$$

$$p_2' = \begin{cases} \frac{1}{2-\lambda_1} & \lambda_1 < \frac{2\kappa-1}{\kappa} \\ 1-\kappa+\kappa\lambda_1 & \frac{2\kappa-1}{\kappa} < \lambda_1 < \kappa \end{cases}$$

$$t' = \begin{cases} \frac{1}{2-\lambda_1} & \lambda_1 < \frac{2\kappa-1}{\kappa} \\ \kappa & \frac{2\kappa-1}{\kappa} < \lambda_1 < \kappa \end{cases}$$

$$\pi' = \begin{cases} \frac{1}{2}\left(\frac{1}{2-\lambda_1}\right)n & \lambda_1 < \frac{2\kappa-1}{\kappa} \\ \frac{1}{2}\left(\lambda_1 - \frac{2\kappa-1}{\kappa} + \frac{1}{\kappa}\right)\kappa^2 n & \frac{2\kappa-1}{\kappa} < \lambda_1 < \kappa \end{cases}$$

if $\kappa < 1/2$, then reselling is the optimal strategy,

$$q_1' = n_1$$

$$p_2' = \frac{n - q + \frac{q}{n}n_1}{n} = 1 - \kappa + \kappa\lambda_1$$

$$t' = \kappa$$

$$\pi' = \frac{1}{2}\left(\lambda_1 - \frac{2\kappa - 1}{\kappa} + \frac{1}{\kappa}\right)\kappa^2 n$$

In all reselling situations, the seller sells as much as possible in period 1, $q_1 = n_1$, and then the seller's remaining inventory is cleared in period 2. The period 2 price is greater than the period 1 price. The seller profit is increasing in the amount of period 1 demand, λ_1

Proof of Theorem 5. The revenue generated from transaction fee is

$$\pi_t^* = \lambda_1\kappa\left(p_2^* - \kappa\right) = \lambda_1\kappa\left(1 - 2\kappa + \kappa\lambda_1\right)n$$

and total revenue is

$$
\begin{aligned}
\pi^* &= p_1^* \lambda_1 n + p_2^* (\kappa - \lambda_1) n + \pi_t^* \\
&= \frac{1}{2} \kappa \left(2 - (2 - \lambda_1) \kappa\right).
\end{aligned}
$$

Therefore,

$$
\frac{\pi_t^*}{\pi^*} = \frac{2\lambda_1 \left(1 - (2 - \lambda_1) \kappa\right)}{2 - (2 - \lambda_1) \kappa}.
$$

Taking the derivative,

$$
\frac{\partial \left(\pi_t^* / \pi^*\right)}{\partial \kappa} = -\frac{2\lambda_1 (2 - \lambda_1)}{\left(2 - (2 - \lambda_1) \kappa\right)^2} < 0.
$$

$\lim_{\kappa \to 0} \pi_t^* / \pi^* = \lambda_1$. Finally, for κ large enough, $\pi^* = \pi_a$ and there is no reselling and therefore $\pi_t = 0$. $\square$